Part Three

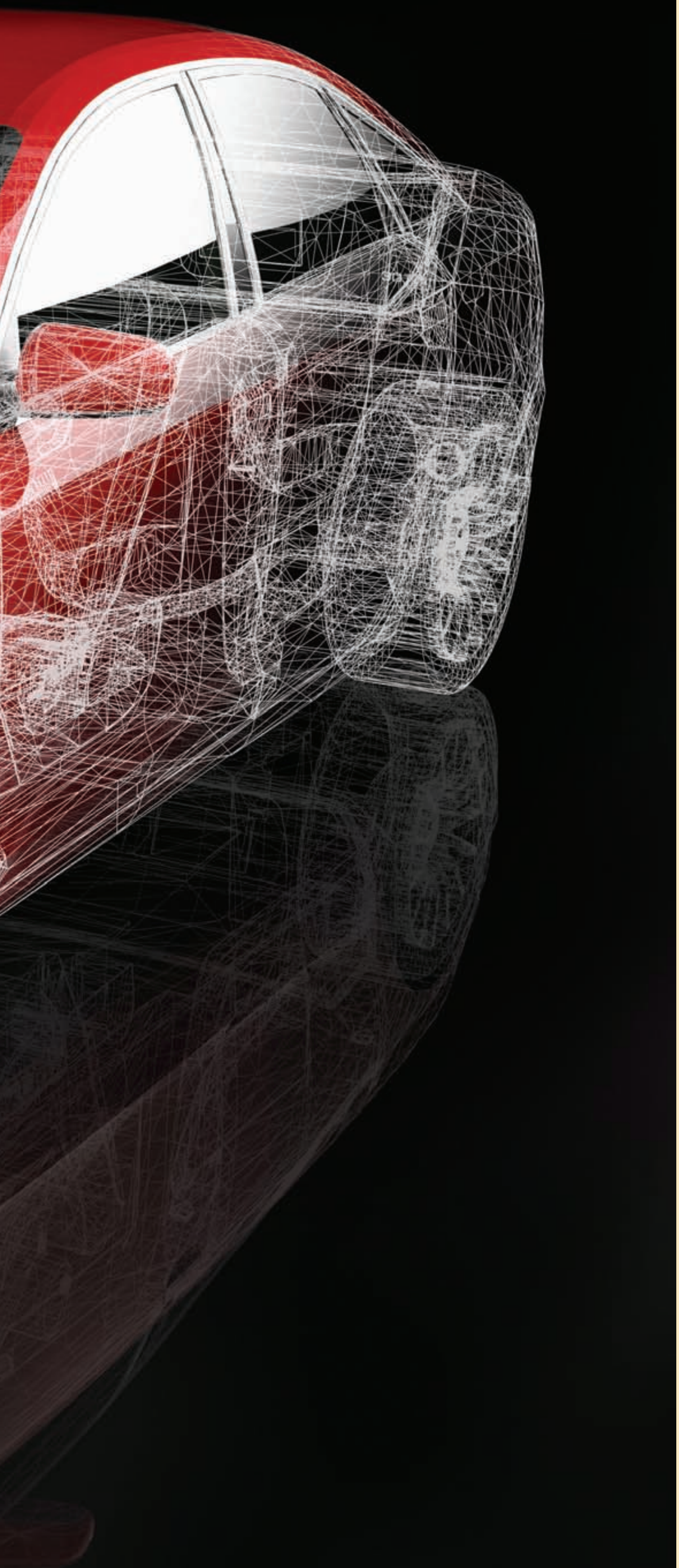
Descriptive Geometry

Descriptive geometry is the mathematical foundation of engineering graphics. Part 3 provides the basics of descriptive geometry, including the important concepts of true-length lines and true size and shape surfaces, as well as the relationships between lines and planes. Part 3 also expands on the multiview drawing concepts defined in Part 2 by describing the types, uses, and construction of auxiliary views.

Finally, Part 3 introduces the essential concepts of intersections and developments. Intersections are created when two geometric forms cross each other. An example would be two intersecting cylindrical pipes in a plumbing system. Developments are created when the surfaces of geometric forms are unfolded or laid out on a flat plane. Examples of developments include (1) the layout of ductwork sections for a heating, ventilation, and air-conditioning system before the sections are cut, bent, and attached to each other to form the complete system, and (2) the layout of an aircraft fuselage on large, flat, sheet metal sections before it is cut out, bent, and attached to the frame.

Both intersections and developments rely on descriptive geometry and auxiliary views for their construction, thus demonstrating the application of technical graphics fundamentals to real-world requirements.

Part Three



Chapter Fourteen

Fundamentals of Descriptive Geometry

Chapter Fourteen

Nothing great can be accomplished without enthusiasm.

—Ralph Waldo Emerson

Objectives

After completing this chapter, you will be able to:

1. Define the theoretical principles of descriptive geometry.
2. Identify and define the direct view, revolution, and fold-line methods.
3. Identify, define, and create principal and nonprincipal lines and planes in various spatial locations.
4. Define and create a true-length view and point view of a line by the auxiliary method.
5. Identify, define, and create parallel, intersecting, and perpendicular lines.
6. Construct true-length lines in a plane.
7. Construct an edge view and true-size view of a plane by the auxiliary view method.
8. Determine the angle between a line and a plane and between two planes.

Introduction

This chapter describes the concepts of descriptive geometry as applied to solving spatial problems. The basic geometric elements

Figure 14.1

Application for descriptive geometry

The design of a chemical plant uses descriptive geometry methods to solve spatial problems.

(© Photri, Inc.)

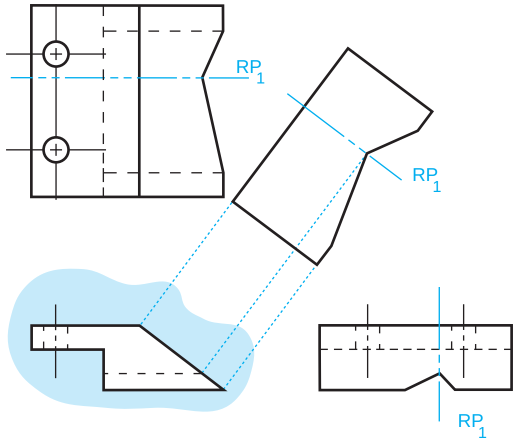


Figure 14.2

Direct view method

of points, lines, and planes, used extensively in traditional descriptive geometry applications, are defined by example problems. These 2-D geometric elements can be combined with 3-D geometric primitives to solve design problems, utilizing 3-D CAD and solid modeling software. Figure 14.1 illustrates the application of descriptive geometry concepts in the design of a chemical plant. For the plant to function safely, pipes must be placed to intersect correctly and to clear each other by a specified distance, and they must correctly intersect the walls of buildings. Descriptive geometry is used to solve these types of spatial problems.

14.1 Descriptive Geometry Methods

Descriptive geometry is the graphic representation of the plane, solid, and analytical geometry used to describe real or imagined technical devices and objects. It is the science of graphic representation in engineering design that forms the foundation, or grammar, of technical drawings.

French mathematician Gaspard Monge (1746–1818) organized and developed the "science" of descriptive geometry in the late 1700s. Monge used orthographic projection and revolution methods to solve design problems associated with complex star-shaped military fortifications in France. Shortly thereafter, in both France and Germany, descriptive geometry became a required part of each country's national education programs. In 1816, Claude Crozet introduced descriptive geometry into the curriculum of West Point in the United States, and in 1821 he published *Treatise on Descriptive Geometry,* the first important English writing on descriptive geometry.

Until CAD technology was developed and widely adopted as a graphics tool, three traditional methods were used to solve spatial design problems through the application of descriptive geometry: the direct view method, the revolution method, and the fold-line method. In certain cases, one method may be preferred over another because of its ease in determining the problem solution. However, all three methods are acceptable.

The direct view method is consistent with the use of 3-D solid modeling to solve spatial problems. In cases where another method gives an equally effective or more direct solution to a problem, both methods are presented. Figure 14.2 illustrates the **direct view method** to find the true size of the inclined plane (view 1). The direct view method, sometimes referred to as the *natural method,* places the observer at an infinite distance from the object, with the observer's line of sight perpendicular to the geometry (i.e., the object's major features) in question.

Figure 14.3 illustrates the **revolution method** of finding the true size of the inclined plane. In this method, the geometry, such as a point, line, plane, or entire object, is revolved about an axis until that geometry is parallel to a plane of projection that will show the true shape of the revolved geometry. In the figure, an oblique face has been revolved to bring it parallel to the profile plane, in which the true shape of that surface is shown. The revolution method is satisfactory when a single entity or simple

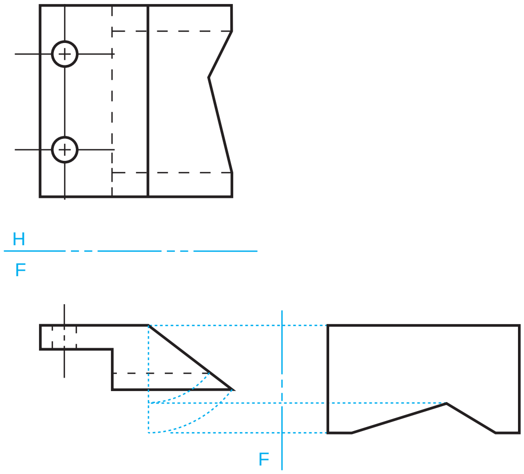


Figure 14.3

Revolution method

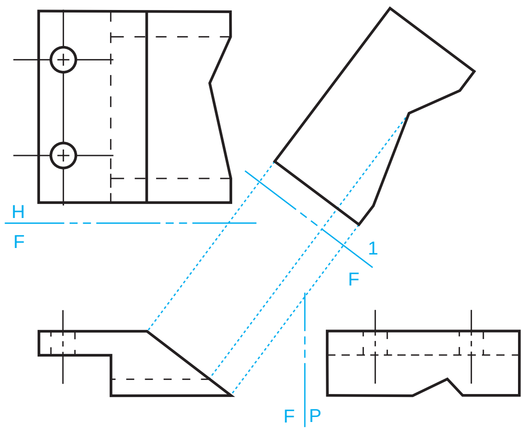


Figure 14.4

Fold-line method

geometry is revolved; however, when the geometry is complex, this method tends to be confusing.

Figure 14.4 illustrates the **fold-line method** of finding the true size of the inclined plane. The fold-line method is also referred to as the *glass box method,* which was discussed in Chapters 7, 10, and 13. Using the fold-line method, the inclined face is projected onto an auxiliary plane that is parallel to the face. The difference between this method and the revolution method is that, in the revolution method, the geometry is revolved to one of the principal projection planes, while the fold-line method uses an auxiliary plane parallel to the desired geometry. The auxiliary projection plane is then "unfolded" so that it is perpendicular to the line of sight.

CAD technology, specifically 3-D solid modeling, allows the use of both the revolution and direct view methods. The revolution method is the underlying principle for the "translation of geometry through rotation" capabilities of CAD. It is assumed that the principal projection plane to which the geometry revolves is the computer screen.

CAD and solid modeling software also utilize the principles of the direct view method. The user can specify the view desired, and the CAD system will position the line of sight perpendicular to the geometry in question. The geometry is consequently parallel to the projection plane (i.e., the computer screen).

14.2 Reference Planes

Because of the practical applications to 3-D CAD and solid modeling, the direct view method is used here to describe the various applications of descriptive geometry. Although there are differences between the direct view method and the fold-line method, either will work for solving spatial design problems that require the use of descriptive geometry.

In Figure 14.5 on the next page, a **reference plane** AA is illustrated. In Figure 14.5A, the reference plane AA is placed on the back of the object. This would place the edge view of the reference plane at the rear of the top and profile views in the multiview drawing. All measurements are taken from this reference plane to specific points on the object in the multiview. Points 1 through 6 can be measured perpendicular from reference plane AA in either the top or profile view by measuring along the Z axis.

The reference plane can be placed anywhere with respect to the object. In Figure 14.5B, it is placed in front of the object, which places it between the horizontal and front views in the multiview drawing. All measurements are taken from this reference plane to specific points on the object in the front and profile views. In the position shown, the reference plane placed between the horizontal and front views is labeled H–F; a reference plane between the frontal and profile planes would be labeled F–P, following the conventions discussed in Chapter 13.

Normally, the edge view of a reference plane is represented by a phantom line consisting of two short dashes, a long dash, and two short dashes, in that order. The use of a reference plane between the views allows for the use of either the direct view or fold-line method for solving descriptive geometry problems.

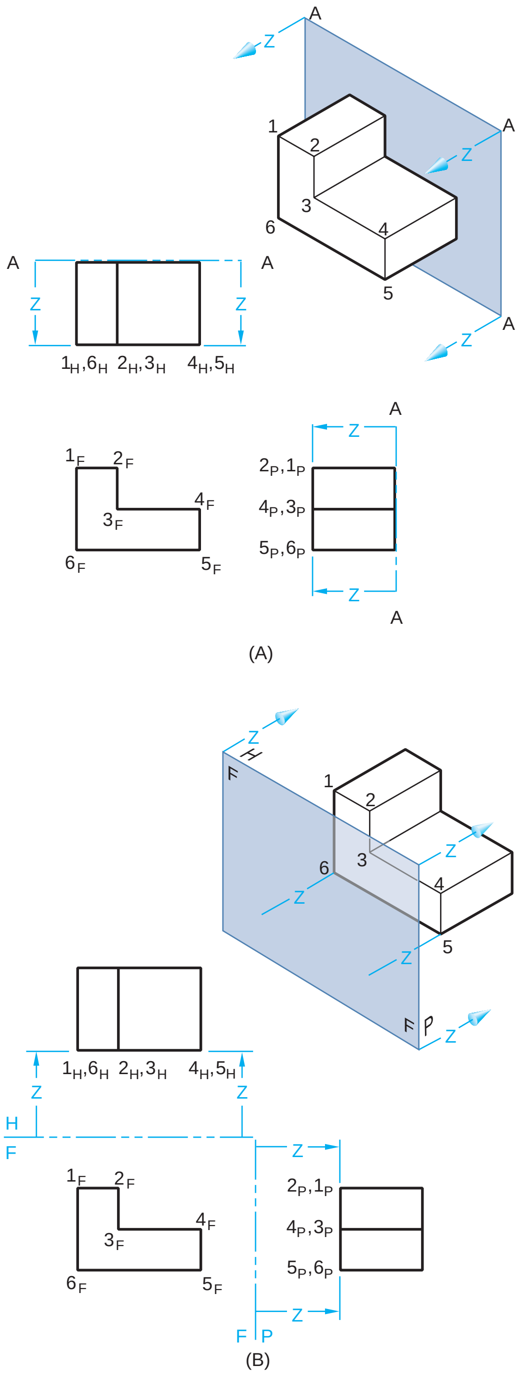


Figure 14.5

Using reference planes

The subscripts used with each number in the multiviews indicate which view the plane is in: F indicates front, H indicates horizontal or top, and P indicates profile or side view.

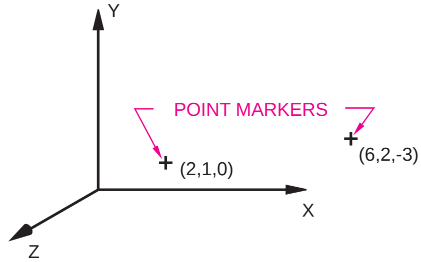


Figure 14.6

Graphical representation of a point

Practice Exercise 14.1

For this Practice Exercise, do Problem 14.1 located at the end of this chapter.

14.3 Points

A *point* has no width, height, or depth. A point represents a specific position in space, as well as the end view of a line or the intersection of two lines. The position of a point is marked by a small symmetrical cross and is located using Cartesian coordinates (Figure 14.6).

14.4 The Coordinate System

Before a graphical solution of a problem can be attempted, the relative locations of two or more points must be described verbally or graphically from a known reference. One of the most generally used reference systems is the *Cartesian coordinate system,* which was described extensively earlier in this text. (See Section 8.3.) The Cartesian coordinate system is the basis for both traditional and CAD systems. Any point can be located by describing its location with respect to the three axes X,Y,Z (Figure 14.7).

With the Cartesian coordinate system, points are located with respect to the origin (0,0,0) and to each other. For example, in Figure 14.8 point S is four inches to the right, three inches above, and two inches in front of the origin (4,3,2). Using orthographic projection, in the top view, point S is located four inches to the right and two inches in front of the origin, but the height cannot be determined. In the front view, point S is four inches to the right and three inches above the origin. In the right side

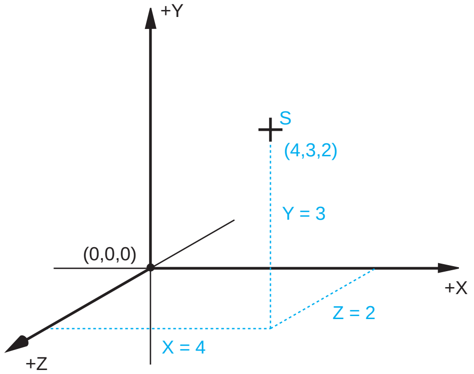


Figure 14.7

Cartesian coordinate system

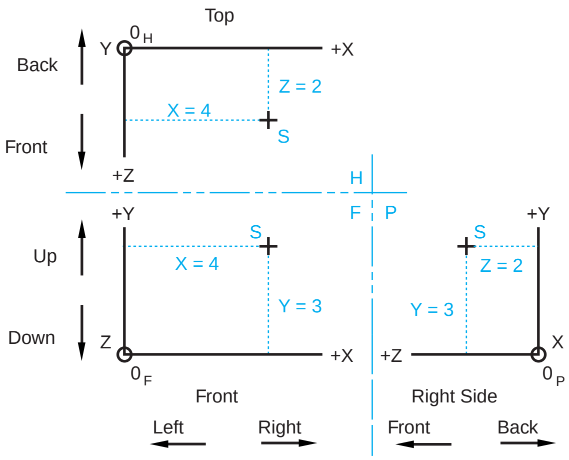


Figure 14.8

Cartesian coordinates in a multiview drawing

view, point S can be located by projecting it from the top and front views or by locating it from the origin.

Thus, in any view adjacent to the front view, movement toward the front view will always be moving from the back to the front of the object. Movement in the front view toward the right side view will always be moving from the left to the right side of the object, and movement toward the top view in the front view will always be moving from the bottom to the top of the object (Figure 14.9). These concepts are important when solving spatial geometry problems using descriptive geometry.

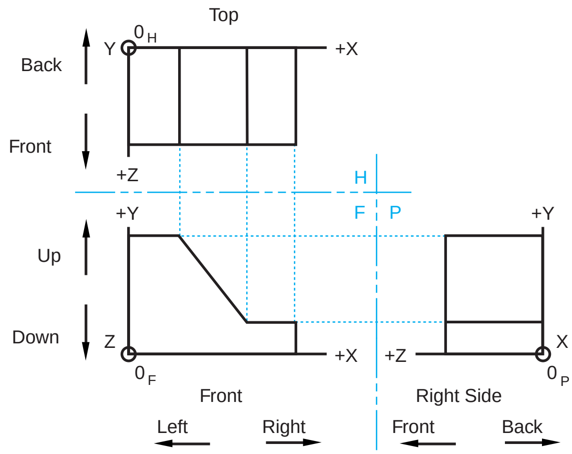


Figure 14.9

Orthographic description using direction

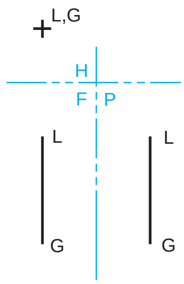


Figure 14.10

Multiview representation of a vertical line

14.5 Lines

A *line* is a geometric primitive that has no thickness, only length and direction. A line can graphically represent the intersection of two surfaces, the edge view of a surface, or the limiting element of a surface.

Lines are either *vertical, horizontal, oblique,* or *inclined.* A vertical line is defined as a line that is perpendicular to the plane of the earth (horizontal plane). For example, in Figure 14.10, line LG is a point in the top view and, in the front and side views, point L is directly above point G; therefore, line LG is a vertical line.

A line is defined as horizontal if all points are at the same height or elevation. Figure 14.11 on page 696 shows three different instances of line LG being horizontal.

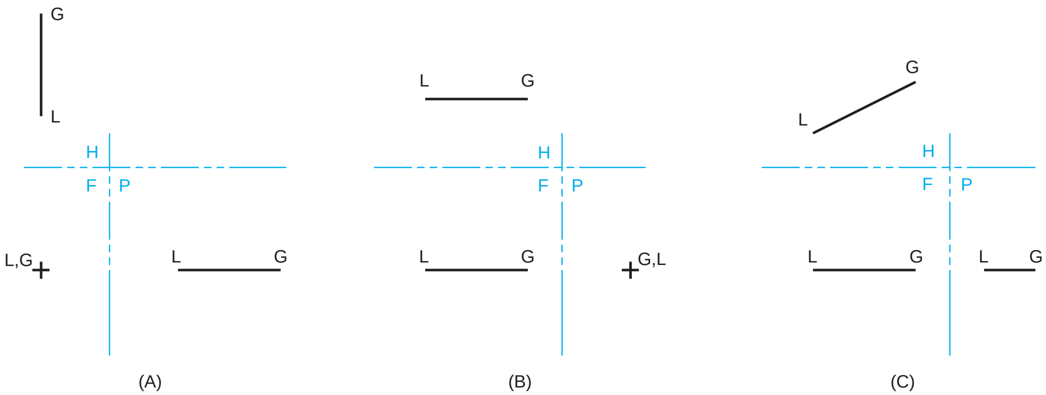


Figure 14.11

Multiview representation of horizontal lines

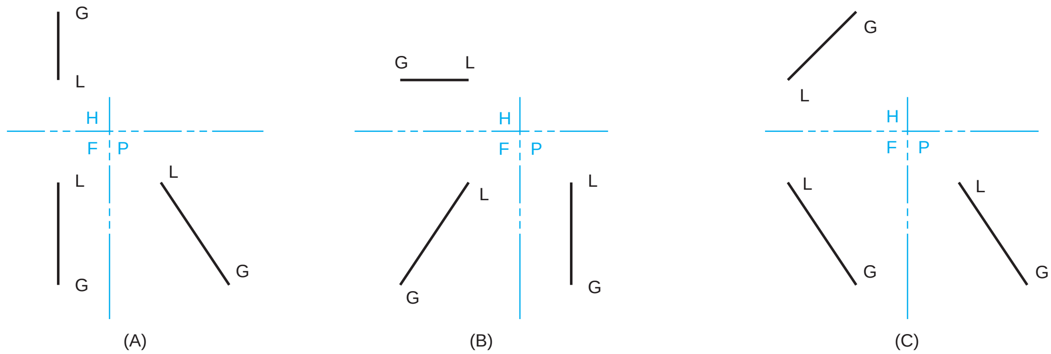


Figure 14.12

Multiview representation of inclined lines

Although a line may be horizontal, its position with respect to the horizontal plane, as shown in the top view, can vary. For both Figures 14.11A and 14.11B, the positions of the line are such that it is a point in either the front or side view. The position of the line in Figure 14.11C describes the general case where a line lies in a horizontal plane and appears true size in the top view.

An inclined line has one end higher than the other (but not at 90 degrees). An inclined line can only be described as inclined in a front or side view; in the top view, it can appear in a variety of positions. Figures 14.12A and 14.12B show different instances of line LG as an inclined line. The positions of the line in Figures 14.12A and 14.12B are special positions in that the line is also parallel to the projection plane, and Figure 14.12C shows the general case of an inclined line known as an oblique line.

A **true-length line** is the actual straight-line distance between two points. In orthographic projection, a true-length line must be parallel to a projection plane. For example, line LG is true length (TL) in the top view of Figure 14.13.

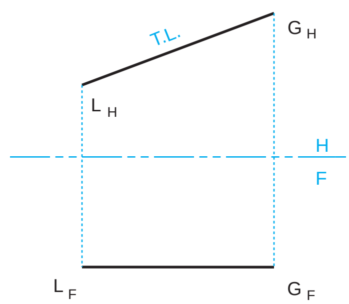


Figure 14.13

Multiview representation of a true-length line

A line can also appear as a point, called an *end* or **point view**. This occurs when the line of sight is parallel to a true-length line. The point view of a line cannot be orthographically determined without first drawing the line in a true-length position (Figure 14.14).

A **principal line** is parallel to one of the three principal projection planes. A line that is not parallel to any of the three principal planes is called an **oblique line** and will appear foreshortened in any of these planes.

A **frontal line** is a principal line that is parallel to, and therefore true length in, a frontal plane. A frontal line can be viewed true length from either a front or back view (Figure 14.15A). A **horizontal line** is a principal line that is parallel to, and therefore true length in, a horizontal plane. A horizontal line can be viewed true length from either a top or bottom view (Figure 14.15B). A **profile line** is a principal line that is parallel to, and therefore true length in, a profile plane. A profile line can be viewed true length from either a left or right side view (Figure 14.15C).

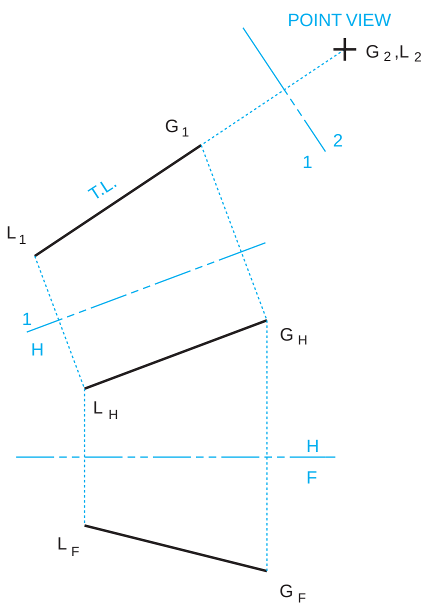


Figure 14.14

Point view of a line

14.5.1 Spatial Location of a Line

The spatial location of a line must be determined before a graphical representation of the line can be drawn. Lines can be located graphically by any one of three different methods: Cartesian coordinates, polar coordinates, or world coordinates. These coordinate systems are discussed in detail in Section 8.3.

14.5.2 Point on a Line

If a point lies on a line, it must appear as a point on that line in all views of the line. Figure 14.16 on the next page illustrates that point X lies on the line LG, but points Y and Z do not. This can be verified by looking in the top view, where point Z is in front of the line LG, and in the front view, where point Y is above the line LG.

14.5.3 True Length of a Line

A line will appear true length in a view if the line is parallel to the plane of projection and the line of sight is perpendicular to the projection plane. Confirmation of parallelism must be obtained by viewing the projection plane

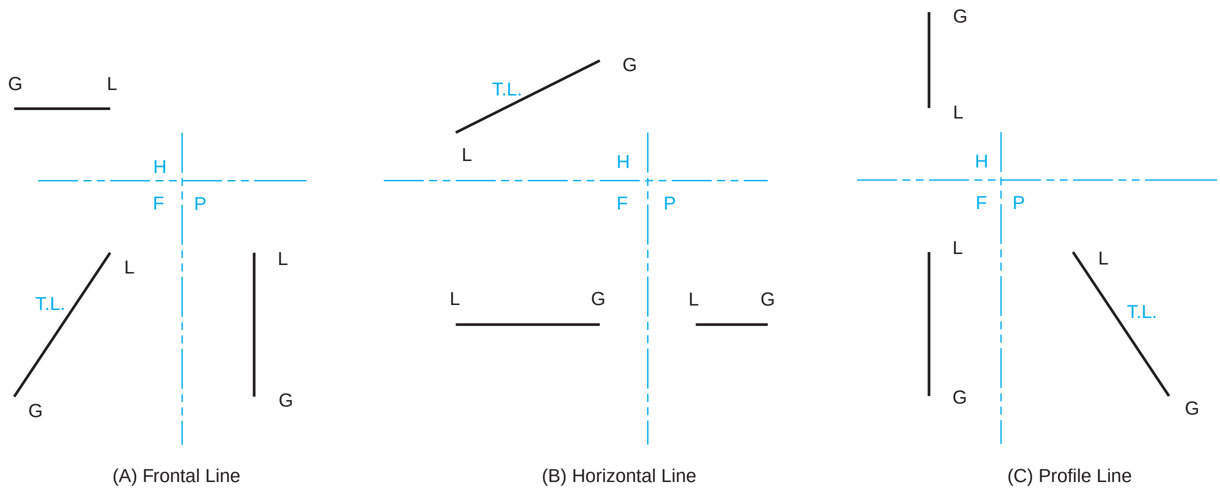


Figure 14.15

Principal lines: frontal, horizontal, and profile

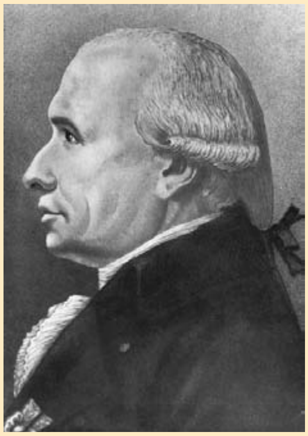
Historical Highlight

Gaspard Monge

Gaspard Monge, the man known as "the father of descriptive geometry," was an incredibly talented man. He was a mathematician, a scientist, and an educator working in physics, chemistry, analytical geometry, and, of course, descriptive geometry.

Monge was born the son of a poor merchant in eighteenth century France. His father managed to help him get a sufficient education and Monge's talent took him the rest of the way. More specifically, while he was a student at Mezeres, his unique solution to a problem involving a fortress design got him promoted from assistant to a full professor. A couple years later he was made a professor of mathematics and later took over the physics department.

Unfortunately, Monge was not permitted to make his unique solution publicly known; it was deemed a military secret. He continued to work on the principles he had used, and expanded and revised them so that they could be used to help solve any technical graphics problem. Then, in 1794, Monge helped to found the first modern engineering school, Ecole Polytechnique, and was finally able to teach the principles of descriptive geometry. It was during the next year that he published his book *Geometrie Descriptive.* His work would change technical drawings from what were simple pictures into actual plans or engineering drawings.



Source: © Bettman/Corbis

Monge received many awards and honors in his lifetime, but they were all taken away when Louis XVIII came to power. Monge had been very loyal to Napoléon Bonaparte, and when Napoléon lost power Monge's career was ruined. He died in disgrace in 1818. There is a positive note, however. Even after his death, Monge still had students and followers who continued his work, overseeing further editions of *Geometrie Descriptive.*

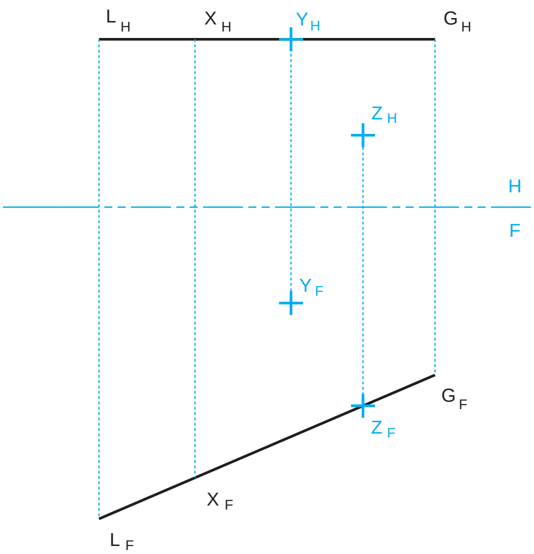


Figure 14.16

Point on a line

in an adjacent view. (See Section 10.4.3.) If the line is indeed parallel to the projection plane, the line will also be parallel to the reference plane's edge view in the adjacent view (Figure 14.17).

Several techniques can be used to find the true length of a line. If the line is parallel to one of the principal projection planes, it can simply be projected onto that plane and then measured. If the line is not parallel to a principal projection plane, then either the auxiliary view or revolution method can be used.

> Principles of Descriptive Geometry Rule 1:
>
> ***True-Length Line***
>
> If a line is positioned parallel to a projection plane and the line of sight is perpendicular to that projection plane, the line will appear as true length.

The following steps describe how to use the auxiliary view method to determine the true length of a line that is not parallel to a principal projection plane.

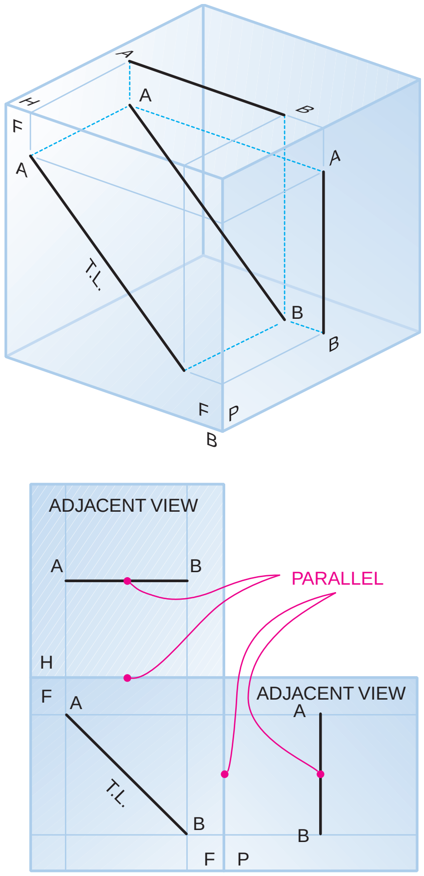


Figure 14.17

True-length line

A true-length line will be parallel to the edge view of the plane in the adjacent views.

True Length of a Line: Auxiliary View Method

Step 1. Draw an auxiliary projection plane parallel to one of the views of the given line (Figure 14.18 on page 700). In the example, the edge view of the auxiliary projection plane is placed parallel to line LG in the front view and is labeled F–1. Construct projection plane H–F between the top and front views, perpendicular to the projectors between the views.

Step 2. From the front view, draw projectors from line LG perpendicular to the auxiliary projection plane.

Step 3. In the top view, measure the vertical distances from points L and G to projection plane H–F.

Step 4. Transfer these distances to the auxiliary view, measuring from the projection plane F–1 and along the projectors from the front view.

Step 5. Connect the new points L and G to form line LG. In this view, line LG is a true-length line that can then be measured directly.

True-length lines are needed when calculating the total length of material needed for the exhaust system on an automobile (Figure 14.19 on page 700).

Revolution Method It is sometimes more practical to find the true length of a line by the revolution method. Figure 14.20 on page 700 illustrates the technique involved. Line LG is revolved around an axis (line LO) that is perpendicular to the line of sight and parallel to a projection plane. Although there are two positions to which line LG can be revolved into its true length, the revolution only alters the spatial position of the line; the size and shape of the line do not change. Regardless of the degree of rotation, the true length of the rotated line will be exactly the same as the true length of the given line. Use the following procedure to develop a true-length line using the revolution method.

True Length of a Line: Revolution Method

Step 1. Given the top and front views of the line LG in Figure 14.21 on page 701, rotate point G into an edge view of a plane parallel to the frontal plane. (Use a compass or measure the length of LG as shown in the top view.)

Step 2. Project the rotated point G from the top view into the front view, using a projector perpendicular to the edge view of the parallel plane.

Step 3. In the front view, draw a projector from G perpendicular to the projector from the top view. This locates the rotated point G in the front view.

Step 4. In the front view, connect point L to the rotated point G to produce true length of line LG.

The true length of a line can be found easily if the line has been drawn on a CAD system. Most CAD software packages have an INQUIRE or similar command that allows you to determine the length or distance of a line by selecting each end of the line. Other information may also be given at the same time, including the angle of the line projected on the X–Y plane and the angle off the X–Y plane. The X, Y, and Z coordinates of each end of the line may also be shown.

CAD can be used to draw a line in a true-length position perpendicular to the line of sight. A user-specified construction plane is selected and a viewpoint is chosen such that the line of sight is perpendicular to that plane.

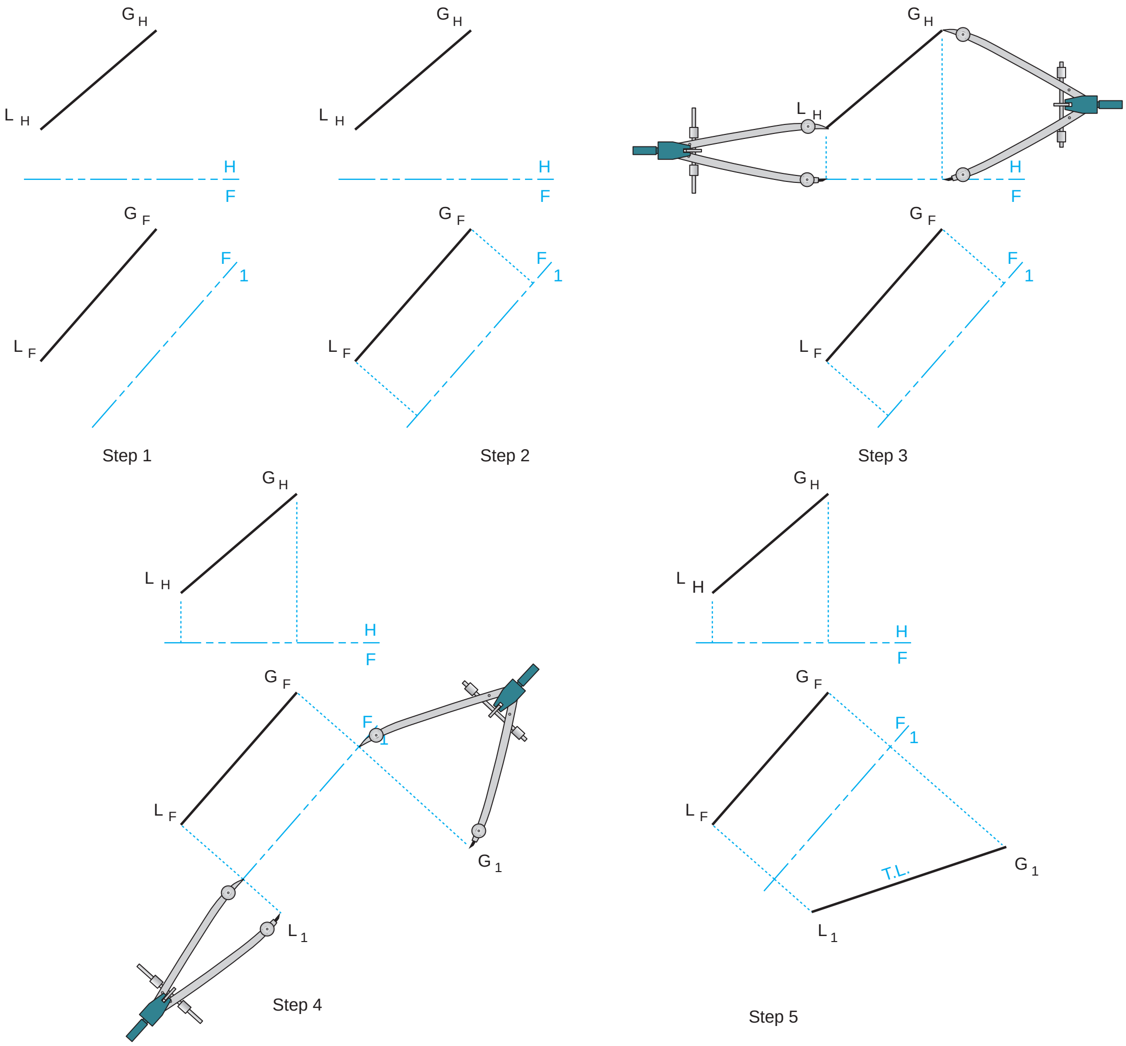


Figure 14.18

True length of a line: auxiliary view method

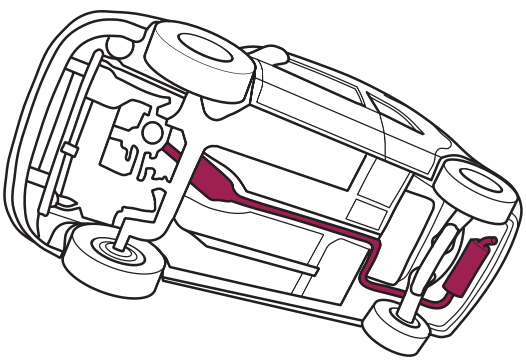


Figure 14.19

Determining the lengths of material for the exhaust system in an automobile is an application for true-length lines.

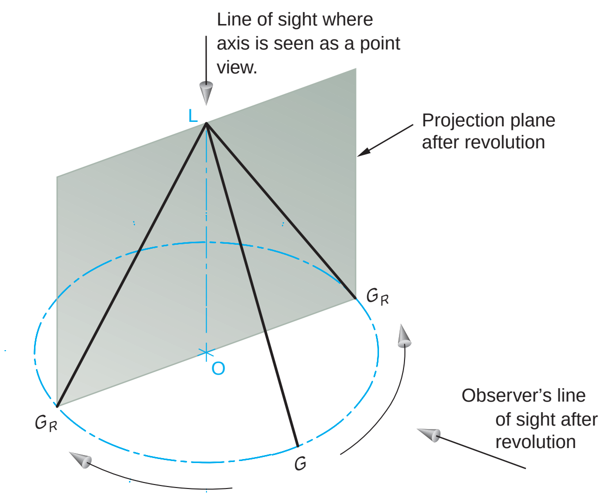


Figure 14.20

Revolution of a line

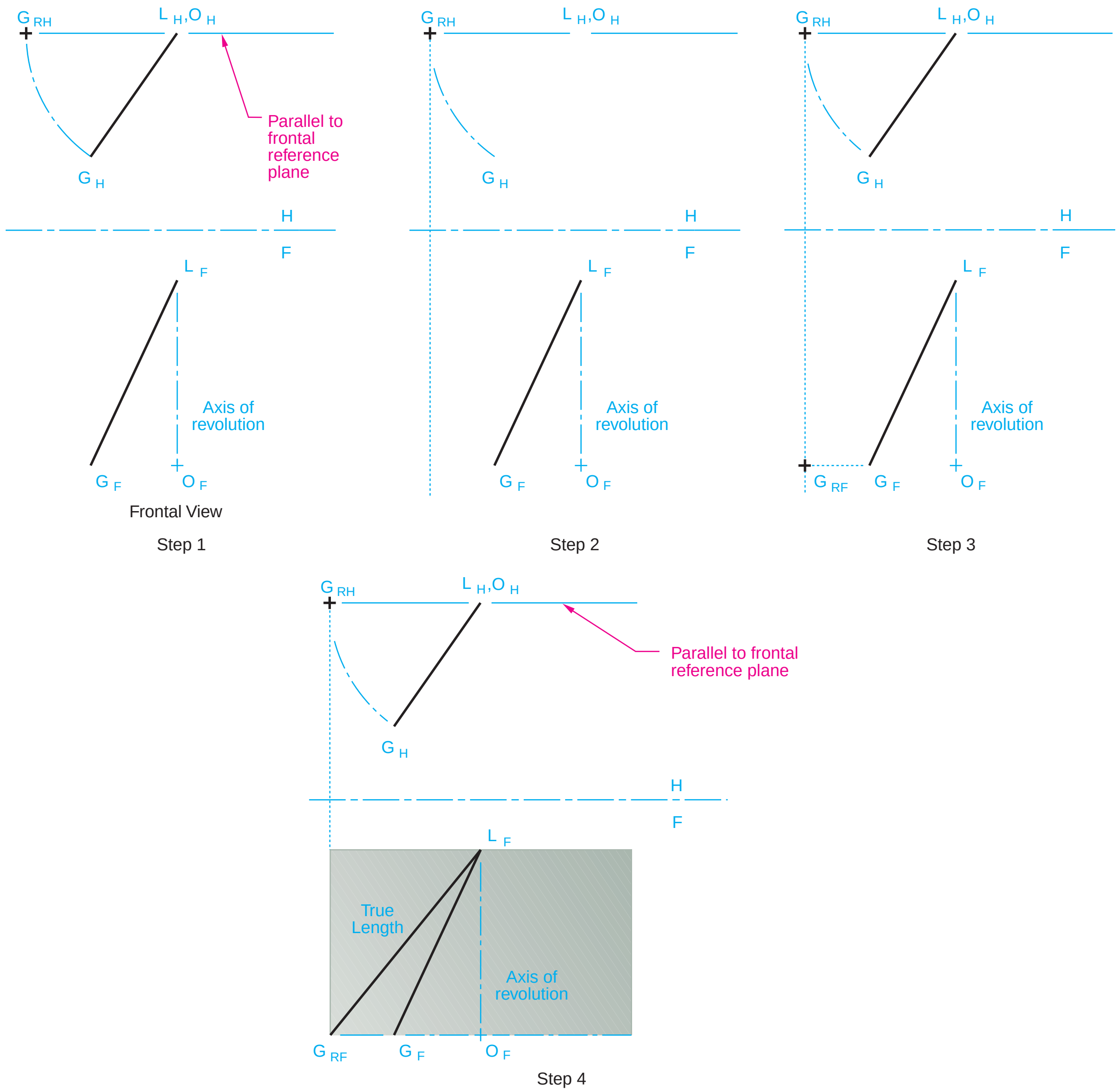


Figure 14.21

True length of a line by revolution

The line is then drawn on the plane. The essential concept is that lines or planes can be drawn in true-length positions on user-specified construction planes that can be located in any spatial position.

14.5.4 Point View of a Line

A point view of a line occurs when the line of sight is parallel to the line. The back end of the line is directly behind the front end. As an example, if the eraser of a pencil is viewed head on, the point of the pencil will not be seen. Other objects appearing as a "point" include the end of a straight piece of pipe, or the axis of a submarine viewed from either the front or back (Figure 14.22 on the next page).

Point View of a Line: Auxiliary View Method

Step 1. Given the top, front, and true-length auxiliary views of a line in Figure 14.22, construct the edge view of reference plane 1–2 perpendicular to the true-length view of line LG. Construct a line parallel to the true-length view of line LG and from point G.

Step 2. In the front view, measure the perpendicular distances from points L or G to reference plane F–1. Transfer this distance to auxiliary view 1–2, measuring from reference plane 1–2, to establish the point view (G,L) of line LG.

In orthographic projection, the point view of a line is found in a view adjacent to the true-length line view. In Figure 14.23, in the front view, the point view of the line is labeled L,G because L is closer to the viewer than G, the opposite end of the line.

In CAD, a point view of a line can be obtained by placing the line parallel to one axis in a user-specified construction plane and selecting a viewpoint that shows the axis as a point. Because the line is parallel to that axis, the desired line will also appear as a point, and the user-specified construction plane will appear as an edge (Figure 14.24).

> Principles of Descriptive Geometry Rule 2:
>
> ***Point View of a Line***
>
> If the line of sight is parallel to a true-length line, the line will appear as a point view in the adjacent view. Corollary: Any adjacent view of a point view of a line will show the true length of the line.

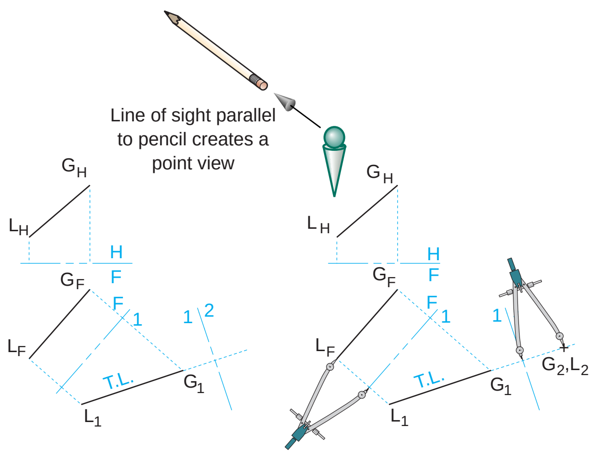


Figure 14.22

Point view of a line, auxiliary view method

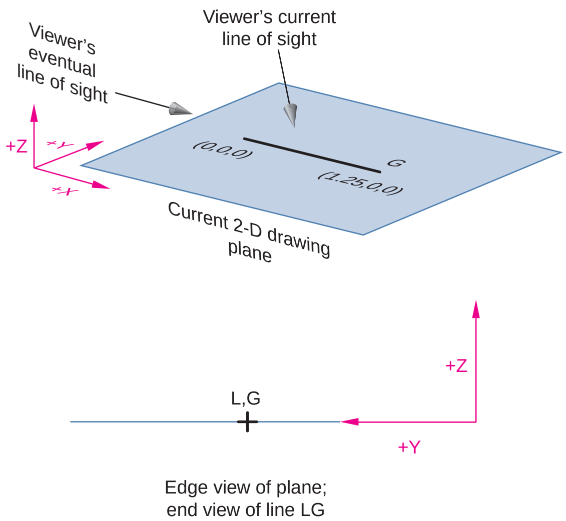


Figure 14.24

CAD application of a point view of a line

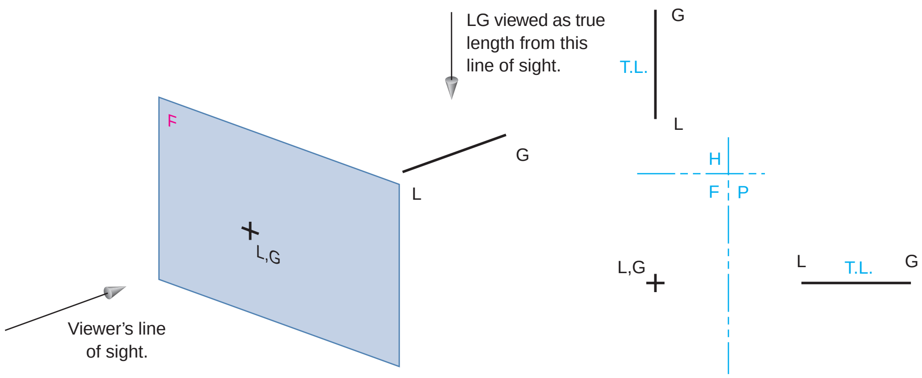


Figure 14.23

Line appearing as a point

14.6 Planes

A plane is positioned by locating any three points on the plane that are *not* in a straight line. Theoretically, planes are limitless; in actual practice, planes are bounded by straight or curved lines. Graphically, planes can be represented in four ways: (1) two intersecting lines, (2) two parallel lines, (3) three points not in a straight line, or (4) a point and a line (Figure 14.25).

In orthographic projection, planes can be positioned such that they appear as (1) an edge, (2) true size and shape, or (3) foreshortened. Planes that are parallel to a principal projection plane are called principal planes and appear true size and shape. A plane not parallel to a principal projection plane but perpendicular to one or more is called an inclined plane and appears foreshortened in at least one principal plane.

14.6.1 Principal Views of Planes

Planes are classified as horizontal, vertical, oblique, or inclined. Figure 14.26 on the following page shows the seven primary positions of the plane CKBU with respect to the three principal views. A **horizontal plane** always appears in true size and shape (TSP) in the top and bottom views, and as an edge in the front, back, and profile views (Figure 14.26A).

A **vertical plane**, as shown in Figure 14.26B, C, and D, always appears as an edge in the top and bottom views. However, it may appear in several ways in the front and profile views. In Figure 14.26B, the vertical plane appears as an edge in the front view and is parallel to the profile projection plane. In this position, the vertical plane is called a **profile plane**. A vertical plane can also be parallel to the frontal plane and appear as an edge in the profile view. This is called a **frontal plane** (Figure 14.26C). Figure 14.26D illustrates a vertical plane that is not a frontal or profile plane, but it is still vertical because it appears as an edge in the top view.

An **inclined plane** is perpendicular but not parallel to a principal projection plane (Figure 14.26D, E, F). An inclined plane never appears true size and shape in a principal view. A plane that appears as a surface, but is not TSP, in all three principal views is called an **oblique plane** (Figure 14.26G).

In the example, the plane always appears as either a four-sided surface or an edge. Any view of a plane must appear as either a surface or an edge. Any surface view of a plane must have a similar configuration as the plane

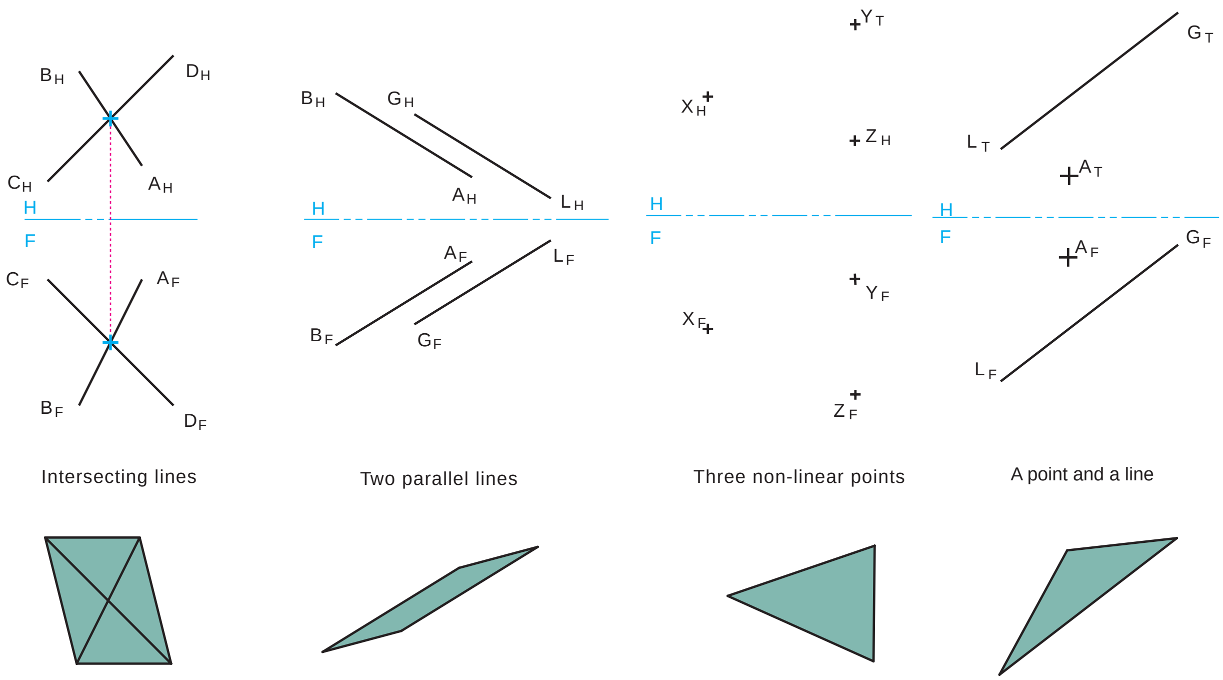


Figure 14.25

Graphical representation of planes

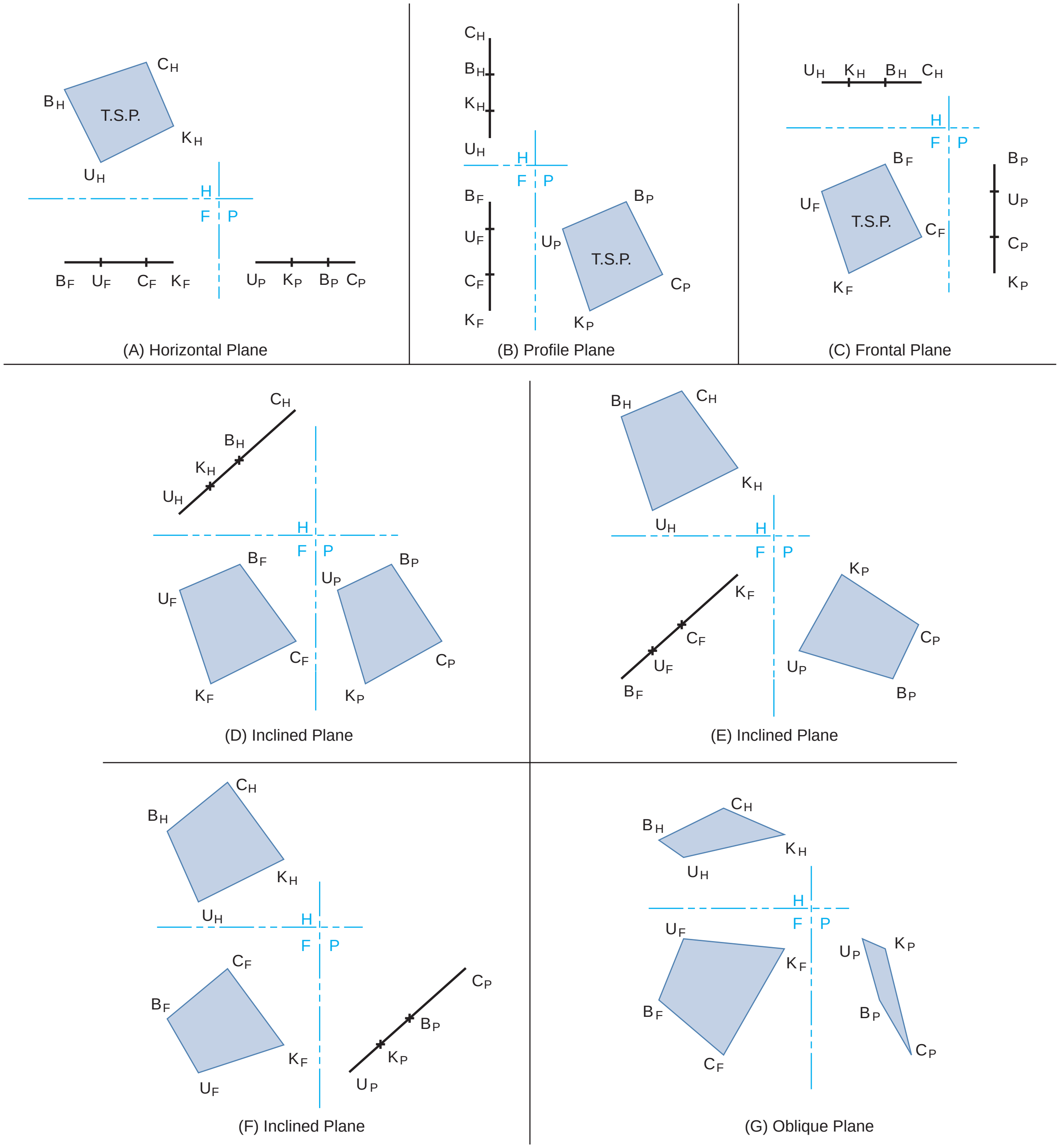


Figure 14.26

Common positions of planar surfaces

(i.e., the same number of sides and a similar shape) (Figure 14.27).

> Principles of Descriptive Geometry Rule 3:
>
> ***Planar Surface Views***
>
> Planar surfaces of any shape always appear either as edges or as surfaces of similar configuration.

14.6.2 Edge View of a Plane

The **edge view of a plane** occurs when the line of sight is parallel to the plane.

> Principles of Descriptive Geometry Rule 4:
>
> ***Edge View of a Plane***
>
> If a line in a plane appears as a point, the plane appears as an edge.

An oblique plane does not appear as an edge in any principal view; therefore, a new view of the oblique plane must be constructed showing the oblique plane as an edge. This new (auxiliary) view is located by determining the view in which a true-length line in the plane will appear as a point.

Edge View of an Oblique Plane: Auxiliary View Method

Step 1. In the front view in Figure 14.28 on page 706, line XA is drawn in the given plane XYZ and parallel to reference plane H–F. Point A is projected into the top view, and true-length line AX is drawn.

Step 2. Auxiliary view 1 is drawn such that the line of sight is parallel to true-length line AX, which will appear as a point. Transferring measurements from the front view, the given plane will appear as an edge in view 1 because point AX is on a line in the plane. The point view of the line is labeled A,X because point A is closest to the observer in that view.

14.6.3 True Size of a Plane

A plane is viewed as true size and shape (normal) if the line of sight is perpendicular to the plane. An estimation of the number of square feet of shingles needed to cover a roof is an application of the *true-size plane* (TSP). Other applications include calculating the square footage of sheet metal or the amount of cardboard needed to construct a shipping container.

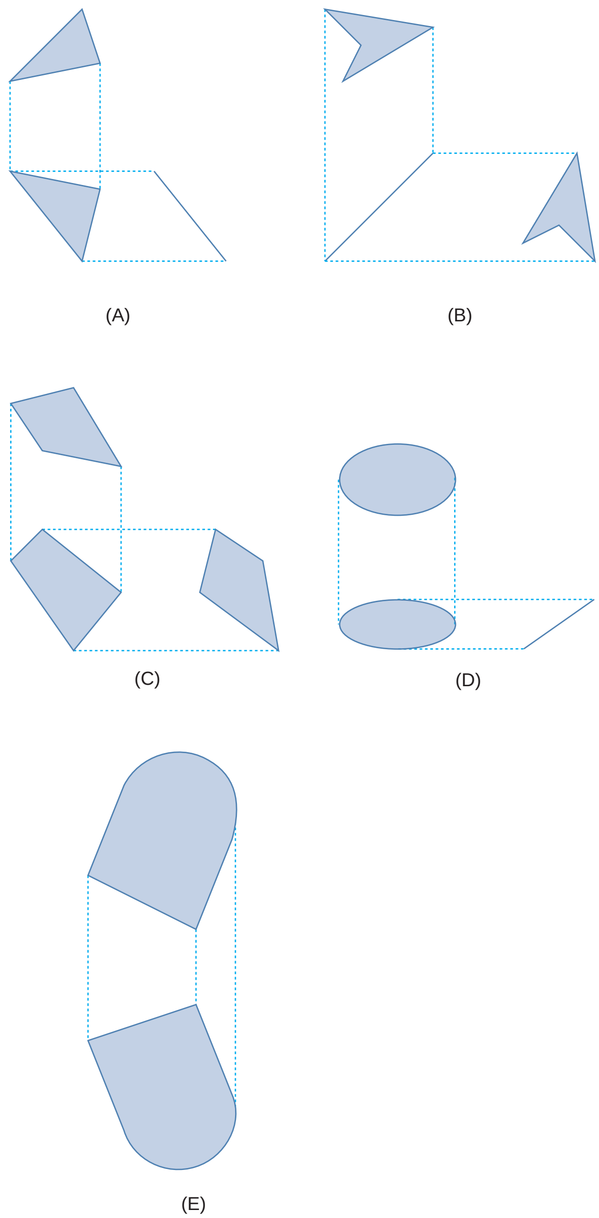


Figure 14.27

Similar configuration of planar surfaces

> Principles of Descriptive Geometry Rule 5:
>
> ***True-Size Plane***
>
> A true-size plane must be perpendicular to the line of sight and must appear as an edge in all adjacent views.

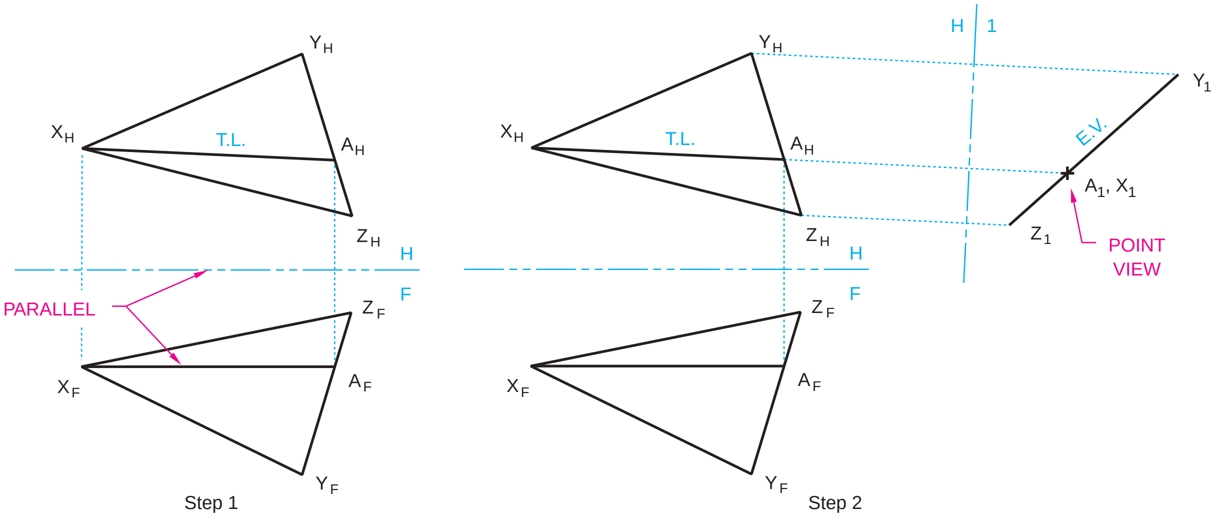


Figure 14.28

Edge view of an oblique plane, auxiliary view method

A line of sight that is perpendicular to the plane can be determined from an edge view of the plane. In Figure 14.29, the true size and shape of plane XYZ is required.

True Size of a Plane: Auxiliary View Method

Step 1. An edge view of the plane is developed in view 1 as previously described.

Step 2. A line of sight perpendicular to the edge view of the plane forms auxiliary view 2, where the plane XYZ is shown as true size and shape. An alternate process could be used to develop auxiliary views 1 and 2 from the top view, to produce the same result.

14.6.4 Angle between Two Planes

To measure the angle between two planes or surfaces, you must construct a view where both planes are on edge.

One example of the angle between surfaces is the windshields on airplanes. Another example is the intersection of the two sloping sections of the roof on a house.

The angle between two intersecting planes is determined in the view in which the line of intersection between the two planes is shown as a point. The solution requires two auxiliary views.

Determining the Angle Between Two Intersecting Planes

In Figure 14.30, planes PLG and LOG intersect at the common line LG.

Step 1. In view 1, a true length of line LG is constructed, along with the complete planes PLG and LOG.

Step 2. In the second view, line LG is shown as point G,L and both planes PLG and LOG appear as edges. The angle between the two planes is then measured.

14.7 Summary

Descriptive geometry is the science of graphical representation and is the foundation of technical drawings. The basic geometric elements for solving spatial problems using descriptive geometry are points, lines, and planes. Most spatial problems are solved by constructing true-length lines, point views of lines, edge views of planes, and true-size views of planes.

The fundamental concepts of descriptive geometry are applicable to both traditional tools and CAD. These fundamentals are in the following tables.

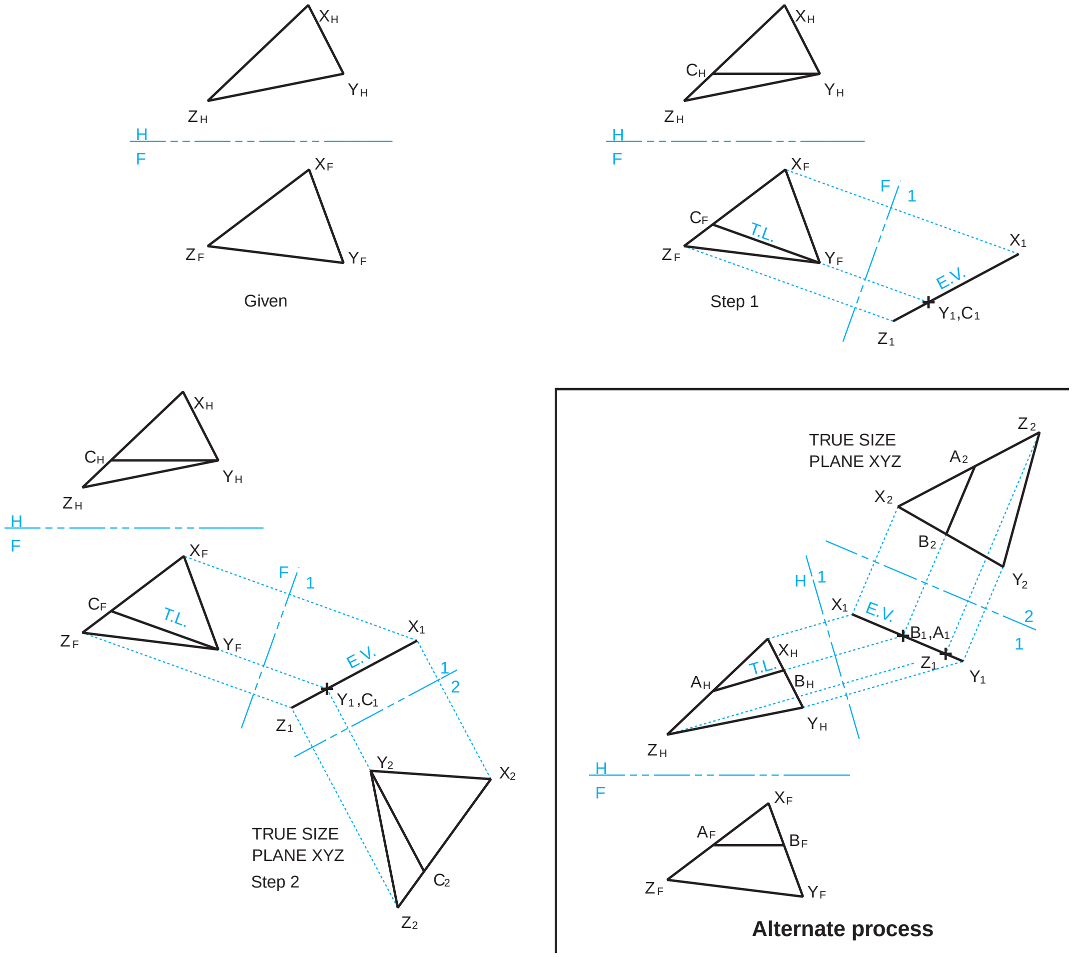


Figure 14.29

True size of a plane, auxiliary view method

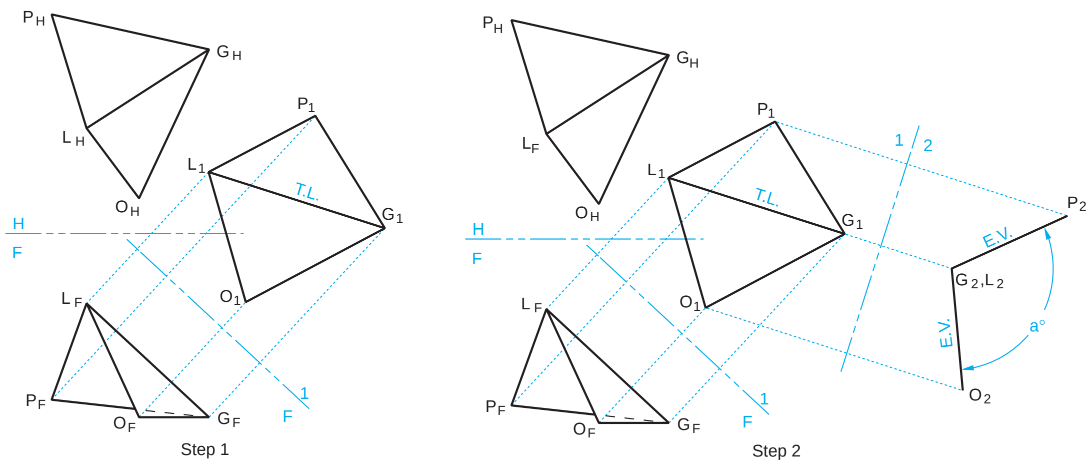


Figure 14.30

Determining the angle between two intersecting planes

Descriptive Geometry Tips

To Find:	Construct:
1. The *true-length view of a line*	A view perpendicular to a given view of that line.
2. The *point view of a line*	The true-length view, then find a view in the direction parallel to that line.
3. The *distance between two lines*	The end view of either line. This view will show the perpendicular distance in true length.
4. The *edge view of a plane*	An end view of any line in the plane.
5. The *shortest distance from a point to a line*	The end view of the line.
6. The *distance from a point to a plane*	The edge view of the plane. Construct a perpendicular line from the point to the plane. This perpendicular line is the true length of the shortest distance.
7. The *true-size (normal) view of a plane*	The edge view of the plane. Then construct a view perpendicular to the edge view.
8. The *angle between two planes*	A view that shows the edge view of both planes. Method 1: Construct an end view of the line of intersection of the planes. Method 2: Construct the normal view of one plane and construct a true-length line on the second plane in this view. Construct a point view of the true-length line.
9. The *angle between a line and a plane*	A view that shows both the true length of the line and the edge view of the plane. Method 1: Construct a normal view of the plane. Then construct a true-length view of the line adjacent to the normal view. Method 2: Construct the point view of the line. Then construct an edge view of the plane adjacent to the end view of the line.

Eleven Descriptive Geometry Facts

If:	Then:
1. A *line* is *horizontal*	It will appear true length in the top view and horizontal in the front and side views.
2. A *line* is *profile*	It will appear true length in the side view and parallel to edge views of profile reference planes.
3. A *line* is *frontal*	It will appear true length in the front view and parallel to edge views of frontal reference planes.
4. A *line* appears as a *point*	Any view adjacent to the point view will be a true-length (normal) view of that line.
5. Two *lines* are *perpendicular*	They will appear perpendicular in a view where one of them appears true length. The other line may be shown either true length, foreshortened, or as a point, depending on its spatial location.
6. A *line* is *perpendicular* to a *plane*	a. It will appear perpendicular to the edge view of the plane. b. It will appear perpendicular to a line in the plane that is true length in the same view. c. A true-length view of that line will also show an edge view of the plane. d. A point view of that line will show a true-size (normal) view of the plane.
7. A *line* is *parallel* to another *line*	They will appear parallel in all views.
8. A *line* is *parallel* to a *plane*	a. An edge view of the plane will show the line parallel to it. b. The line must be parallel to a line in the plane in all views.
9. A *plane* is *parallel* to another *plane*	a. An edge view of one plane will also show an edge view of the other plane. b. A true-size (normal) view of the plane will also show a true-size view of the other plane. c. The planes will intersect a third plane in parallel lines.
10. A *plane* is *perpendicular* to another *plane*	A normal view of one plane will show an edge view of the other plane.
11. Two *lines intersect*	The same point of intersection lies on both of the lines in all views and can be projected between the views.

Online Learning Center (OLC) Features

There are a number of Online Learning Center features listed below that you can use to supplement your text reading to improve your understanding and retention of the material presented in this chapter at www.mhhe .com/bertoline.

- Learning Objectives
- Chapter Outline
- Questions for Review
- Multiple Choice Quiz
- True or False Questions
- Flashcards
- Website Links
- Animations
- Related Readings
- Stapler Design Problem

Questions for Review

1. Define descriptive geometry.
2. Who developed the "science" of descriptive geometry?
3. Describe the difference between the direct view, revolution, and fold-line methods of descriptive geometry.
4. Define a point.
5. Define and describe the Cartesian coordinate system.
6. Define a line.
7. Describe a foreshortened line.
8. Describe a principal line.
9. Describe an oblique line.
10. Describe and define a frontal line.
11. Describe and define a horizontal line.
12. Describe and define a profile line.
13. Describe and define a true-length line using the fold-line method.
14. Describe and define a line appearing as a point.
15. Define a plane.
16. What are the three spatial classifications of a plane?
17. Describe and define the three principal views of a plane.
18. List the steps needed to show a plane as an edge using the auxiliary view method.
19. List the steps needed to show a true size of a plane using the auxiliary view method.

Further Reading

Pare, E. G., R. O. Loving, I. L. Hill, and R. C. Pare. *Descriptive Geometry*. New York: Macmillan Publishing Company, 1991.

Stewart, S. *Applied Descriptive Geometry*. Albany, New York: Delmar Publishing Company, 1986.

Wellman, B. L. *Technical Descriptive Geometry*. New York: McGraw-Hill Publishing Company, 1987.

Problems

14.1 Use a sheet of stiff, clear plastic as a projection plane and a small object containing an inclined surface.

a. Set the object on a table. Hold the projection plane perpendicular to your line of sight and move around the object until you see the inclined surface in its true size and shape. Draw the inclined surface on the projection plane with a water-based marker. This is an example of the direct view method.

- What is the relationship of your line of sight to the projection plane? To the inclined surface? What is the relationship between the projection plane and the inclined surface?
- What is the relationship between the projection plane and the other (normal) surfaces of the object? From this viewpoint, can you see any of the normal surfaces in their true size and shape?

b. Hold a different projection plane out in front of you and position the object so that you are looking at an edge view of the inclined surface. This should be one of the standard views. You should see one or more normal surfaces in true size and shape. Rotate the object 90 degrees so that the inclined surface is seen in its true size and shape. Draw the projection of the inclined surface on the projection plane. This is an example of the revolution method.

- Does the projection you drew look any different from the one you drew in (*a*)? What is the difference between the ways the two views were generated?
- What is the relationship between the projection plane and the normal surfaces of the object? From this viewpoint, can you see any of the normal surfaces in their true size and shape?
- When you rotate the object 90 degrees to see the inclined surface, imagine pivoting the projection plane about an edge on the object. Which edge would you use? Place this edge of the object against the projection plane and pivot the object back and forth between the auxiliary and original standard view.

c. Return the projection plane and object to their locations at the beginning of (*b*). Do not erase the image drawn on the plane. Instead of rotating the object, rotate the projection plane to just above and parallel to the inclined surface. Without changing your viewpoint, try to imagine what you would see if your line of sight were perpendicular to the projection plane. Rotate the plane back to its original position. Your original projection sketch is what you imagined the inclined surface would look like. This is an example of the fold-line method.

- With the projection plane rotated back, draw the standard view next to the inclined surface view you have already drawn. Do any of the new surfaces drawn share edges with the inclined surface? Highlight the edge(s) in contrasting color. What is the relationship of these two views of the edge? Is this the edge about which you pivoted the projection plane in (*b*)?
- Look back at how you generated the two projections of the inclined surface. Do the projections look the same? In which methods did the projection plane move? In which did your point of view change? In which did the object rotate?

14.2 Use the same setup as Problem 14.1, and use a wooden pencil or thin wooden dowel.

a. Place the pencil against the back of the projection plane and draw the projection of the pencil. What is the relationship between the length of the line you drew and the length of the pencil? This is a true-length line.

b. Rotate the pencil so that one end lifts off the projection plane by an inch. Draw the resulting projection in a different color. What is the difference between the two projections? The second projection is a foreshortened view. Lift the end of the pencil more, draw the projection, and compare it with the previous ones.

c. Continue rotating (i.e., lifting one end only, thereby rotating it about the other end) until the projection has reached its minimum length. How long is the projection? What does it look like? How much of the pencil is still touching the projection plane? This is a point view of the line representing the pencil. From this point view of the pencil, rotate the pencil 90 degrees back toward the projection plane. What is the length of the projection now?

d. Figure out two ways you can rotate the pencil without foreshortening the projection. How can you rotate it to create a new true-length projection? How can you rotate it so that it creates exactly the same true-length projection no matter how much you rotate it?

14.3 Repeat the setup in Problem 14.2 using two pencils of equal length. You will need two people for this exercise.

a. Place both pencils against the back of the projection plane. Arrange the pencils so that they are parallel to each other and draw their projections. Are both projections true length?

b. Rotate one of the pencils about an axis perpendicular to the projection plane. The pencils are no longer parallel to each other. Draw their new projections. Are the projections parallel to each other? Are the projections both still true length?

c. Return the pencils to their original parallel arrangement. Rotate one pencil by lifting one end about 1″ off the projection plane. Make sure not to rotate it in any other direction. Draw the projections again. Are the projections parallel to each other? Are the projections both true length?

d. Holding the pencils where they were in (*c*), rotate the projection plane 90 degrees to generate a second standard view. Are the pencils parallel to each other? Rotate the projection plane again and create a third standard view. Are the pencils parallel to each other? In how many views are the projections parallel?

e. Rotate one pencil as you did in (*b*) and the other as you did in (*c*). Are the projections parallel to each other in this view? Are the projections parallel to each other in either of the other two standard views? Is there any view where the projections of both pencils are true length?

14.4 Use a clear plastic projection plane, two triangles made from stiff cardboard, and a pencil. Pierce the middle of one of the triangles with the pencil. Try to keep the pencil perpendicular to the triangle. The pierced triangle is triangle B; the unpierced one is A.

a. Place one edge of triangle A against the back of the projection plane. What is the triangle's relationship to the plane when it is seen as an edge? How far must it be rotated to be seen in true size? What does it look like in incremental degrees of rotation? In which of these different rotations is the edge against the projection plane seen in its true length? Is there a way to orient the triangle such that you see it as an edge view but none of its edges are seen in true length?

b. Repeat (*a*) but with triangle B. What is the relationship of the pencil to the projection plane when the triangle is seen as an edge? Around what axis can you rotate the triangle and still preserve the triangle's edge view? What is the relationship of the pencil to the projection plane when the triangle is seen in its true size? Around what axis can you rotate the triangle and still preserve its true size?

c. Explore possible relationships between the two triangles and the pencil. Will the pencil ever pierce triangle A if A is perpendicular to B? What is the relationship of the pencil to A when the two triangles are parallel to each other? Around what axis can the triangles rotate and stay parallel to each other?

14.5 Using the same items as in Problem 14.4, pierce the pencil through the centers of both triangles A and B. Pivot the triangles about their piercing points so that an edge of each of the triangles can be joined with a piece of tape. This setup will be referred to as a construction in the rest of this problem. The triangles should now form an acute angle with each other. Orient the construction so that the pencil is parallel to the projection plane and the two triangles are seen in edge views. Draw the projection of the two triangles onto the projection plane and note the angle between the two planes.

a. Keeping the pencil parallel to the projection plane, rotate the construction 90 degrees in either direction. Draw two or three intermediate projections of the two triangles, highlighting the angle between the two planes in a different color. What is the relationship between the projected angle in the various views? In which view is the angle seen in true size?

b. Return the construction to the initial edge view. Now rotate the construction 90 degrees until the

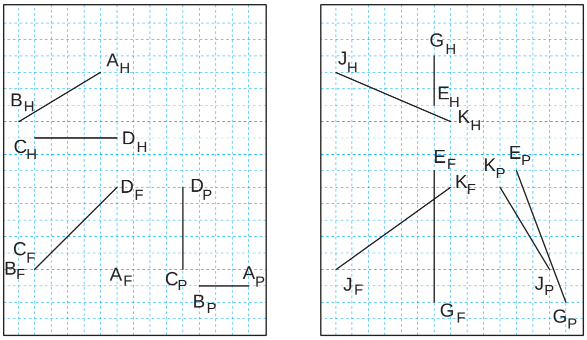


Figure 14.31

Name the types of lines

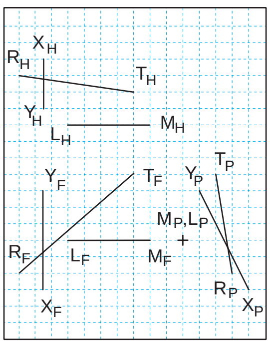


Figure 14.32

Visualization of lines

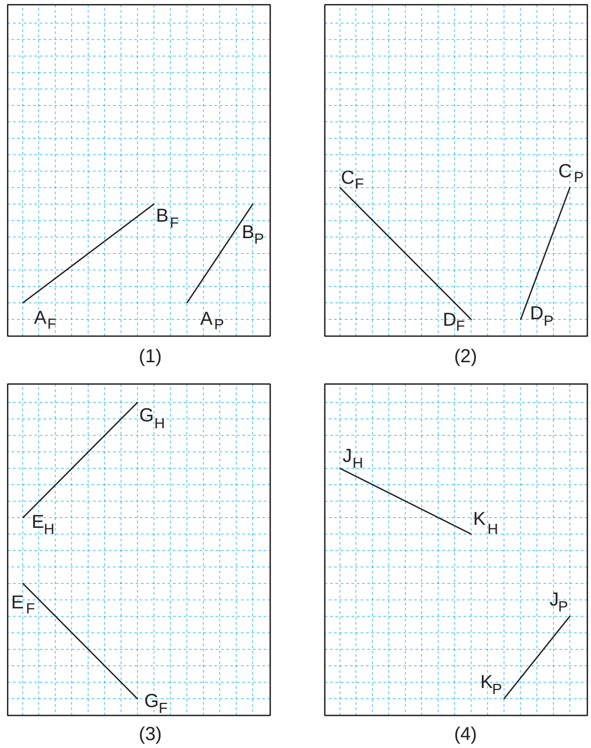


Figure 14.33

Constructing true-length lines

pencil is perpendicular (point view) to the projection plane. Draw two or three intermediate projections of the two triangles, highlighting the angle between the two planes in a different color. What is the relationship between the projected angle in the various views? In which view is it seen in true size? How do these projections compare to the ones drawn in (*a*)?

14.6 In Figure 14.31, name the type of line for AB, CD, EG, and JK.

14.7 (Figure 14.32)

a. Name the type of line for LM.

b. Which point(s) is/are closest to the viewer when looking at the front view?

c. Which point(s) is/are farthest away from the viewer when looking at the front view?

d. Which point(s) is/are highest to the viewer when looking at the front view?

e. Which point(s) is/are lowest to the viewer when looking at the front view?

For the following problems, use an A-size or A3 sheet of paper and draw the given views of the lines or planes before doing the problems. The grid is 0.25″ or 6 mm.

14.8 (Figure 14.33)

a. Draw a true-length view of line AB.

b. Draw a true-length view of line CD.

c. Draw the true-length view of line EG, and measure its length.

d. Complete the front view, and find the true-length view of line JK.

14.9 (Figure 14.34)

a. The true length of line AB is 1.75″ or 44 mm. Point A is in front of point B. Complete the top view.

b. The true length of line CD is 2.00″ or 51 mm. Point D is in front of point C. Complete the top view.

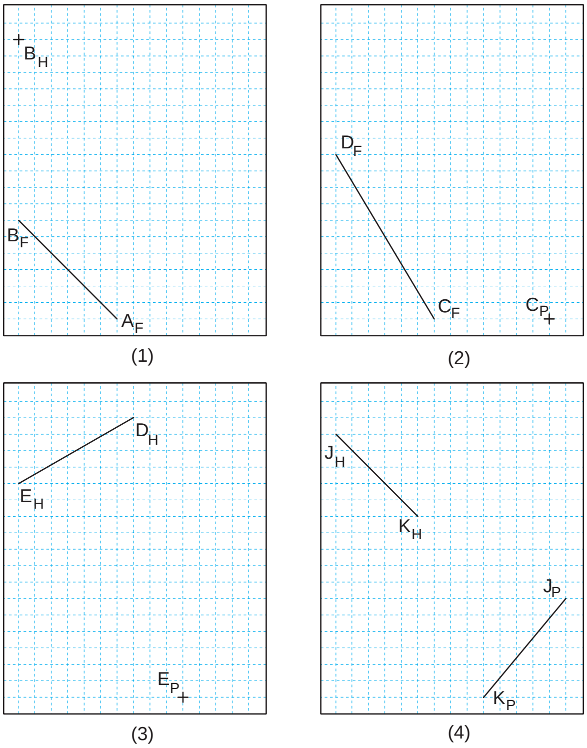


Figure 14.34

Line construction

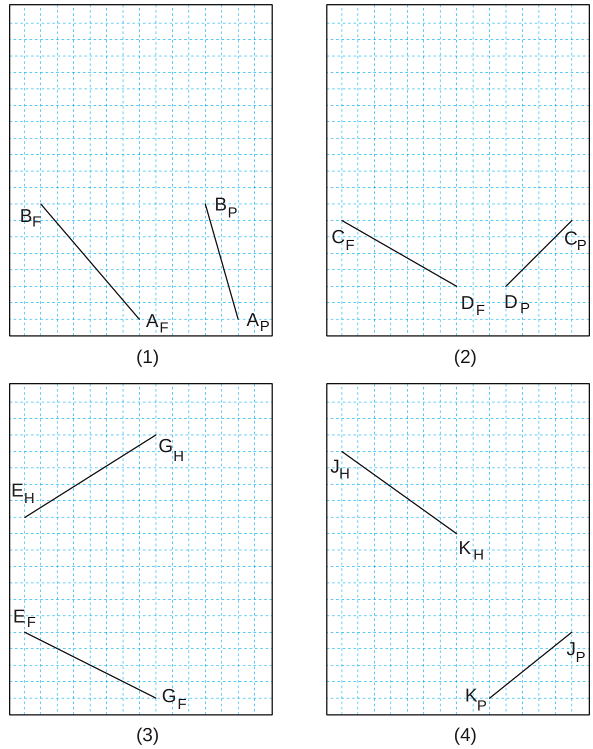


Figure 14.35

Constructing true-length lines

c. The true length of line ED is 1.75″ or 44 mm. Point E is below point D. Complete the front and profile views.

d. The true length of line JK is 1.50″ or 38 mm. Point J is above point K. Complete the front view.

14.10 (Figure 14.35)

a. Draw the true-length view of line AB.

b. Draw the true-length view of line CD.

c. Find the true length of line EG.

d. Complete the front view, and find the true length of line JK.

14.11 (Figure 14.36)

a. Find the true lengths of lines A1, B2, C3, and D4.

b. Find the lateral surface area of the pyramid.

c. You are the commander of a starfleet with three ships, A, B, and C. Each ship has equal top speeds on patrol. An emergency beacon sounds

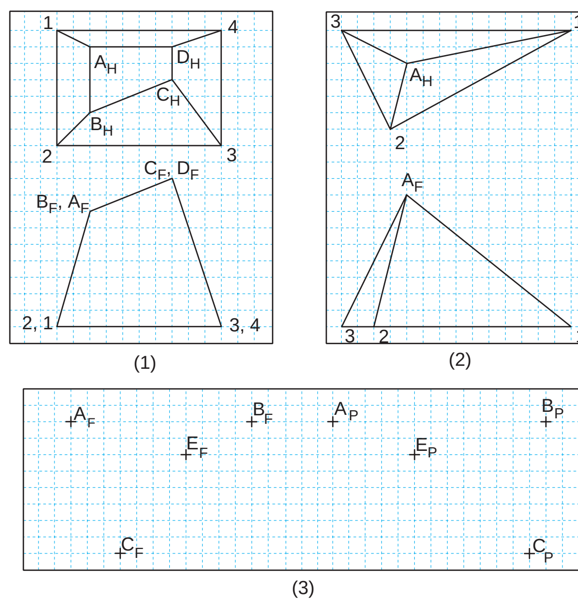


Figure 14.36

Determining true-length distances

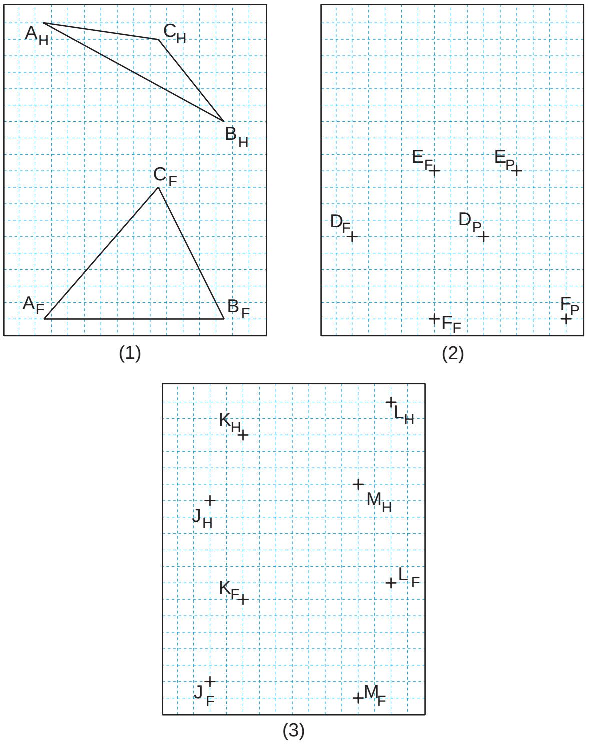


Figure 14.37

Constructing edge views of planes

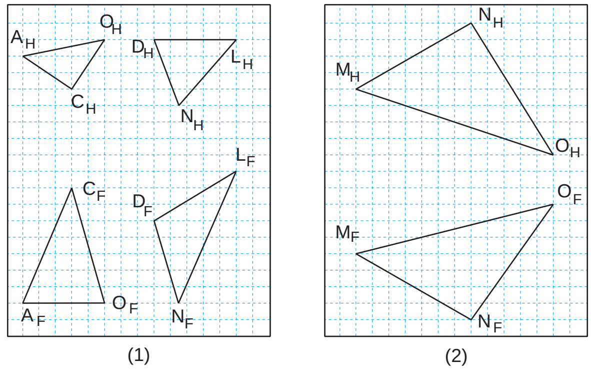


Figure 14.38

Construct true-size views of planes

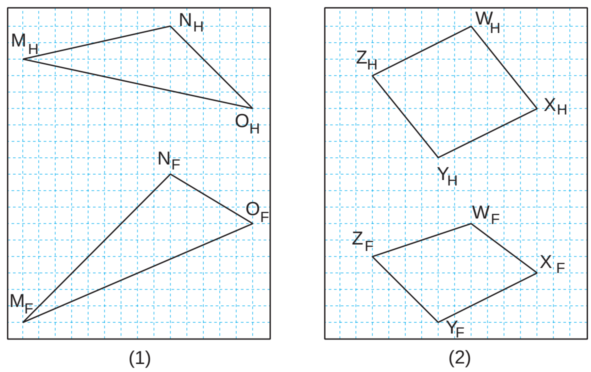


Figure 14.39

Constructing true-size views of planes

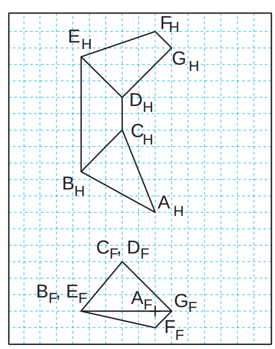


Figure 14.40

Calculating areas

from point E. Which ship is the closest to respond? How far away is each ship? Scale 1″ = 0.5 lightyears.

14.12 (Figure 14.37)

a. Points A, B, and C form a plane in space. Show an edge view of the plane.

b. Points D, E, and F form a plane in space. Show an edge view of the plane.

c. Given points, J, K, L, and M, draw an edge view of the plane formed by the points.

14.13 (Figure 14.38)

a. Draw a true-size view of planes AOC and DLN.

b. Find the area of the largest inscribed circle in the oblique plane MNO. Scale 1″ = 20′.

14.14 (Figure 14.39)

a. Draw a true-size view of oblique plane MNO using the rotation technique.

b. Find the true-size view of plane WXYZ using the rotation technique.

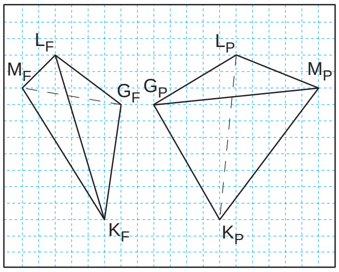


Figure 14.41

Angle between planes

14.15 (Figure 14.40) A protective shield is to be made as shown. Due to the cost involved, an estimate is necessary. How many square feet are in the protective shield? Scale 1″ = 30′.

14.16 (Figure 14.41) Find the dihedral angle between planes LGM and LGK. The object is a tetra-hedron.

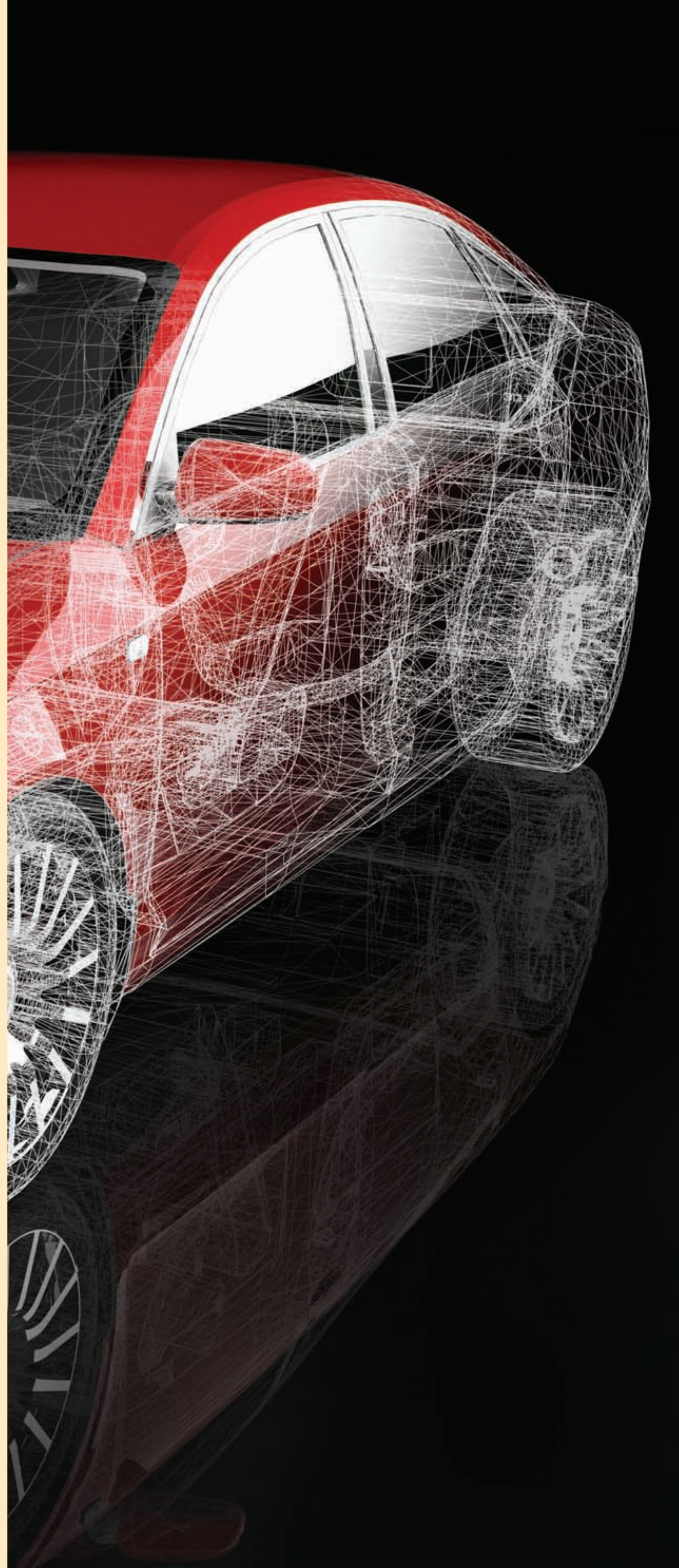
Chapter Fifteen

Chapter Fifteen

Intersections and Developments

The greatest thing in this world is not so much where we are, but in what direction we are moving.

—Oliver Wendell Holmes



Objectives

After completing this chapter, you will be able to:

1. Define and apply the principles of geometric intersections.
2. Identify, define, and apply the intersection of lines and planes using the edge view method.
3. Determine the correct visibility between two surfaces.
4. Identify, define, and create the intersection of a plane and a solid using the cutting plane method.
5. Identify, define, and create the intersection of a plane and a solid using the auxiliary view method.
6. Identify, define, and create the intersection of two planes.
7. Identify, define, and create the intersection of two solids.
8. Define and apply the theoretical principles of geometric developments.
9. Identify and classify the various types of developments.
10. Identify, define, and create the developments of various solids and transition pieces.

Introduction

Intersections and developments are commonly found in many technical disciplines. Two surfaces that meet form a line of intersection. A pipe going through a wall is an example of an intersection. As an engineer or technologist, you must be able to determine the exact point of intersection between the pipe and the wall. A development is the outside surface of a geometric form laid flat. Sheet metal products are an application for developments. A cardboard cereal box, the heating and air conditioning ductwork used in buildings, or large aircraft, when laid flat, are examples of developments. As a technologist, you should be able to visualize and create the development of common shapes, such as cones, prisms, and pyramids. This chapter will address both traditional methods and CAD techniques for drawing intersections and developments.

15.1 Intersections and Developments

Many intersection problems are solved using descriptive geometry techniques described in Chapter 14. Developments are produced by determining the true lengths of the various elements of the form, such as the corners of pyramids or the circumferences of circular features. Some types of surfaces, such as warped surfaces and spheres, can only be developed by approximation.

Intersections and developments are commonly used in a variety of industries. The oil and chemical industries use intersections in the design and construction of refineries (Figure 15.1). Developments are used for the ducts of heating and cooling systems (Figure 15.2) and in the design and production of automobiles and aircraft (Figure 15.3).

Figure 15.1

Intersection application

Intersections are important in the petrochemical industry.

(© Photri, Inc.)

Figure 15.2

Development application

Developments are commonly used in the design of heating, ventilating, and air conditioning (HVAC) systems.

Figure 15.3

Developments are used to manufacture the skin of an aircraft.

15.2 Intersections

An intersection is a point or line where two geometric forms, such as lines or surfaces, meet or cross each other. The type of intersection created depends on the types of geometric forms, which can be two- or three-dimensional. Intersections must be represented on multiview drawings correctly and clearly. For example, when a conical and a cylindrical shape intersect, the type of intersection that occurs depends on their sizes and on the angle of intersection relative to their axes. The line of intersection is determined using auxiliary views and cutting planes.

15.2.1 Correct Multiview Representations: Visibility

Visibility, as used here, is the clear and correct representation of the relative positions of two geometric figures in multiview drawings. In each view, the visibility of the figures is indicated by drawing the figure that is in front (i.e., the most visible) with object lines entirely, while drawing the second figure with both object lines and dashed lines. The dashed lines would be used for those features of the second figure that are covered up or obscured by the first figure and are therefore not visible (hidden) in that view. The concept of visibility is crucial in accurately depicting the intersection of two geometric forms, whether they are two-dimensional or three-dimensional.

The basic concept of visibility can be demonstrated with two skew lines, in which one line is either in front of, behind, above, below, to the left of, or to the right of the other line. In Figure 15.4A, lines LG and AB are skew lines. This is verified by projecting point C from the top view to the front view, which shows that the lines do not intersect at point C. Similarly, the lines do not intersect at point D in the top view.

Because the lines do not intersect, point C can only fall on one of the lines, and either line LG or AB is above the other in the general area of point C. In Figure 15.4B, point E has also been placed at the same location as point C in the top view, and one of these two points must be above the other. Arbitrarily, point C was placed on line AB, and point E was placed on line LG. Both of these points are projected into the front view, which shows that point C is higher than point E. Because point C is higher than point E, it is nearer to the observer in the top view.

In Figure 15.4C, points X and Y appear to be an intersection in the front view, but when the points are projected to the top view, they show that the lines do not intersect. Point X, which is on line LG, is in front of point Y, which lies on line AB; therefore, in the front view, the point is labeled X,Y because X is in front of Y.

The adjacent view procedure can also be used to determine the visibility of two cylinders, such as plumbing pipes or electrical conduit. In Figure 15.5A, both views of the cylinders are incomplete because the visibility has not been determined. In the top view in Figure 15.5B, the apparent intersection of the center lines of the cylinders (point C on line AB and point E on line LG) is projected into the front view. In the front view, point C is revealed to be higher than point E. Therefore, cylinder AB is above cylinder LG. This is graphically represented in the top view by drawing cylinder AB entirely with object lines and drawing that portion of cylinder LG that is behind AB with dashed lines.

The adjacent view procedure can then be used to determine which cylinder is in front of the other (Figure 15.5C).

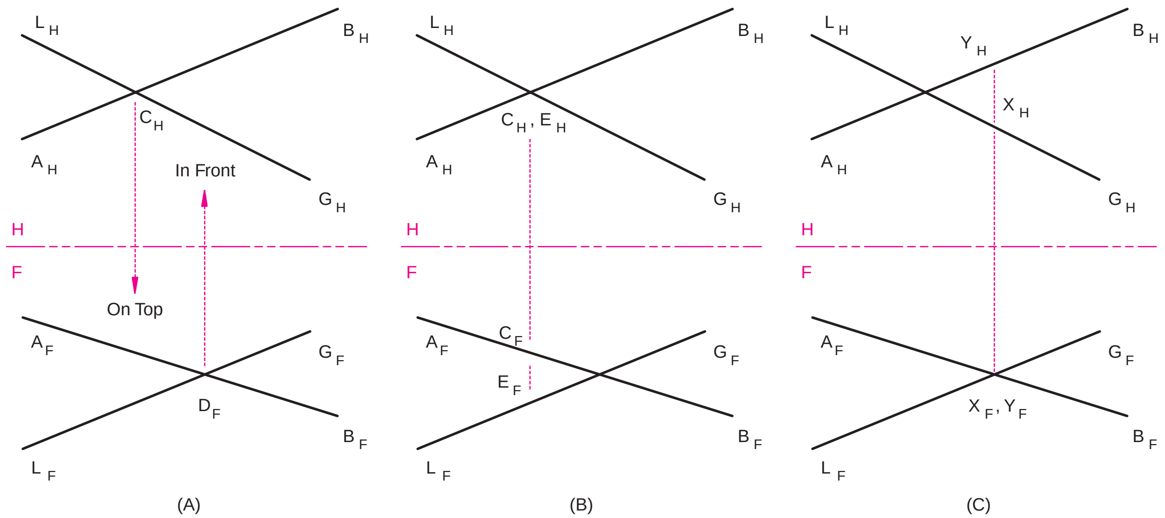


Figure 15.4

Visibility demonstrated by two skew lines

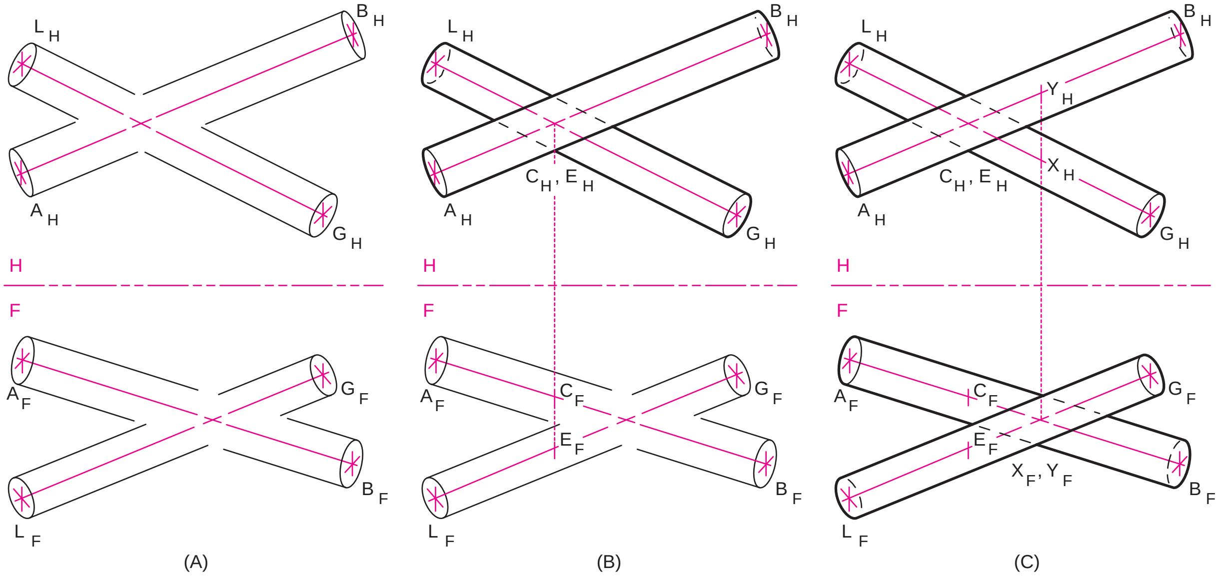


Figure 15.5

Visibility demonstrated by two cylinders

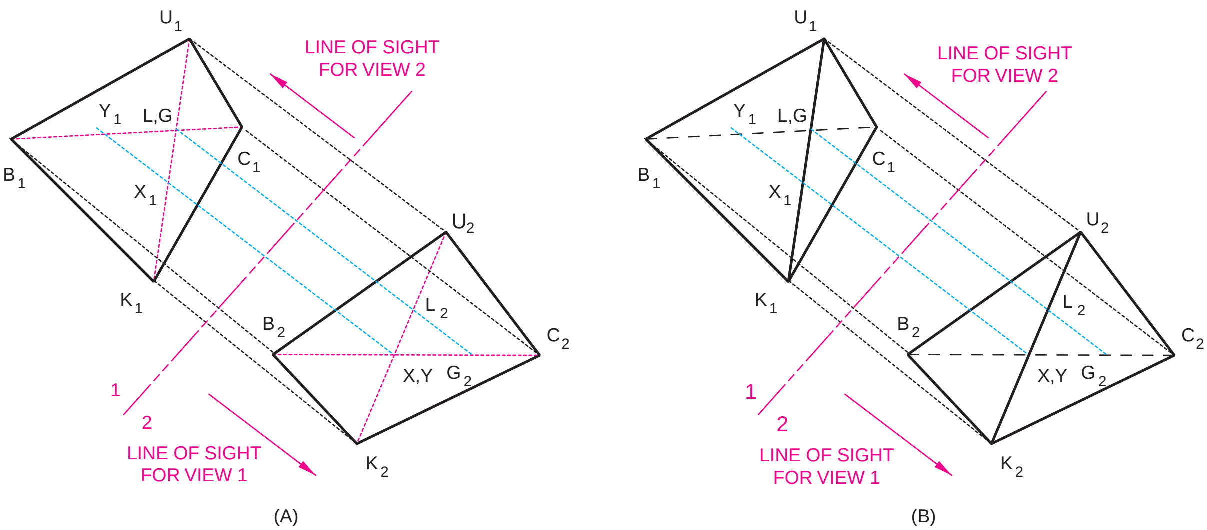
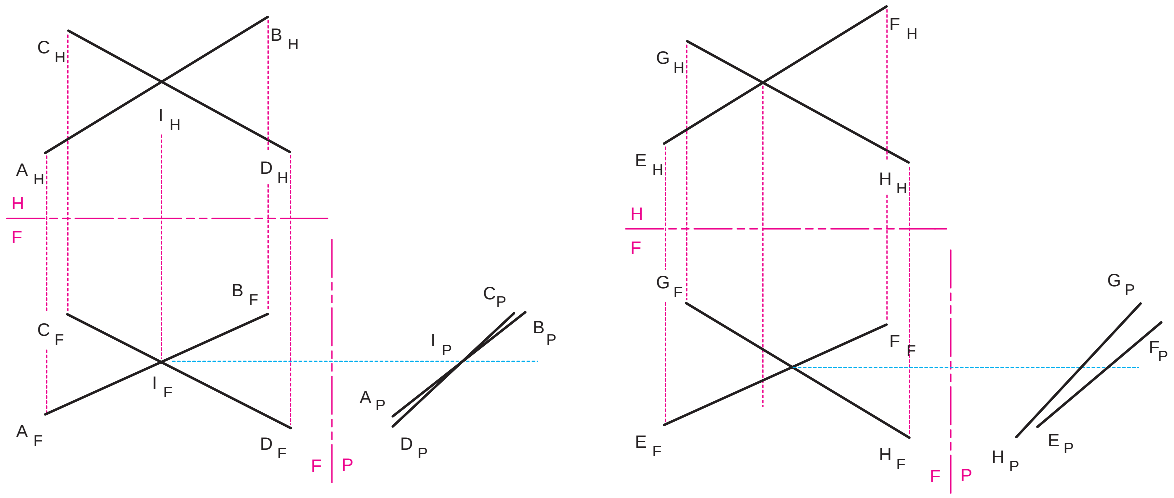


Figure 15.6

Determining visibility between any two adjacent views

In the top view, points X and Y are the apparent intersection of the center lines of cylinders AB and LG. They are projected into the front view, where it is revealed that point X is in front of point Y. This means that cylinder LG is in front of cylinder AB; therefore, in the front view, cylinder LG will be represented entirely by object lines, and cylinder AB will be represented by hidden lines where it is behind cylinder LG.

Any pair of adjacent views can be used to depict visibility. In Figure 15.6A, the visibility of the tetrahedron CKBU must be determined in view 1 and view 2. Because the direction of the line of sight for one adjacent view is



(A) Yes! (B) No!

Figure 15.7

Determining if two lines intersect

always toward the other adjacent view, the visibility of CKBU in view 1 can be determined by looking at view 2. The apparent intersection of lines BC and KU at points L and G is projected into view 2, where it is revealed that point L on line KU is closer to view 1 than point G on line BC; thus, in view 1, line KU is visible and line BC is hidden (Figure 15.6B). To determine the visibility of CKBU in view 2, examine the apparent intersection of lines KU and BC at points X and Y in the adjacent view, which is view 1. View 1 shows that point X on line KU is closer to view 2 than point Y on line BC; thus, in view 2, line KU is visible and line BC is hidden.

15.2.2 Intersection of Two Lines

Two lines that intersect share a common point. In Figure 15.7A, point I is the common point of intersection between lines AB and CD. This is verified by projecting point I to an adjacent view. If the lines do not have a common point that projects from view to view, the lines are nonintersecting (Figure 15.7B). In the profile view, lines GH and EF do not share a common point; therefore, they do not intersect but simply cross in space.

15.2.3 Intersection of a Line and a Plane

The intersection of a line and a plane is referred to as the **piercing point**. A line will intersect a plane if the line is not parallel to the plane. Because lines are usually defined graphically as line segments and because planes are usually bounded, it may be necessary to extend either the line or the plane to determine the intersection point. The two methods used to determine the piercing point between a line and a plane are the edge view method and the cutting plane method.

Edge View Method In the edge view method, the plane is shown as an edge, which means that all points on that plane can be represented on that edge. In Figure 15.8, line LG and plane ABC are given, and the intersection between them must be determined.

Determining the Intersection Between a Line and a Plane: Edge View Method

Step 1. Using methods described in Chapter 14, the edge view of plane ABC is constructed in the first auxiliary view, and line LG is carried into the view. This view reveals that line LG and plane ABC intersect at point Z.

Step 2. Point Z is then projected back into the top and front views. Point Z is always attached to line LG and, in both the top and front views, it falls outside of the bounded plane ABC. Yet, if plane ABC were unbounded or infinite, line LG would intersect the plane at point Z.

Cutting Plane Method The cutting plane method can also be used to find the intersection between a plane and a line. In Figure 15.9, line LG and plane ABC are given, and the intersection between them is to be determined.

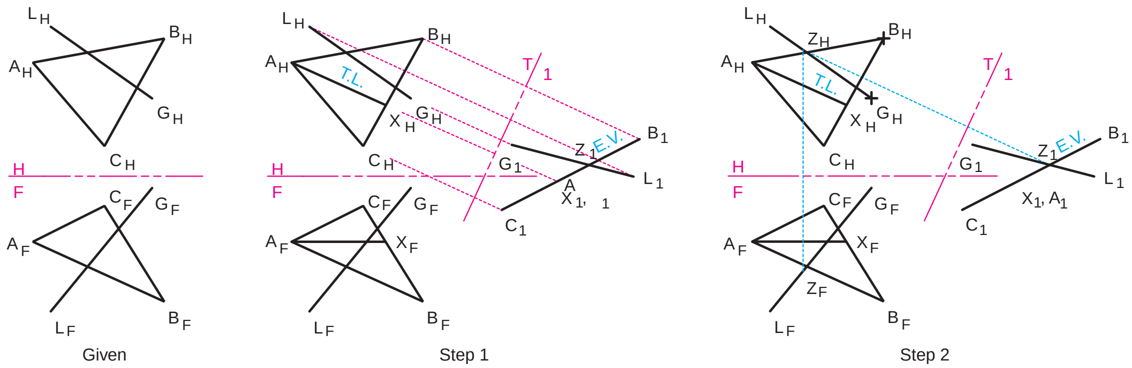


Figure 15.8

Determining the intersection or piercing point between a line and a plane, edge view method

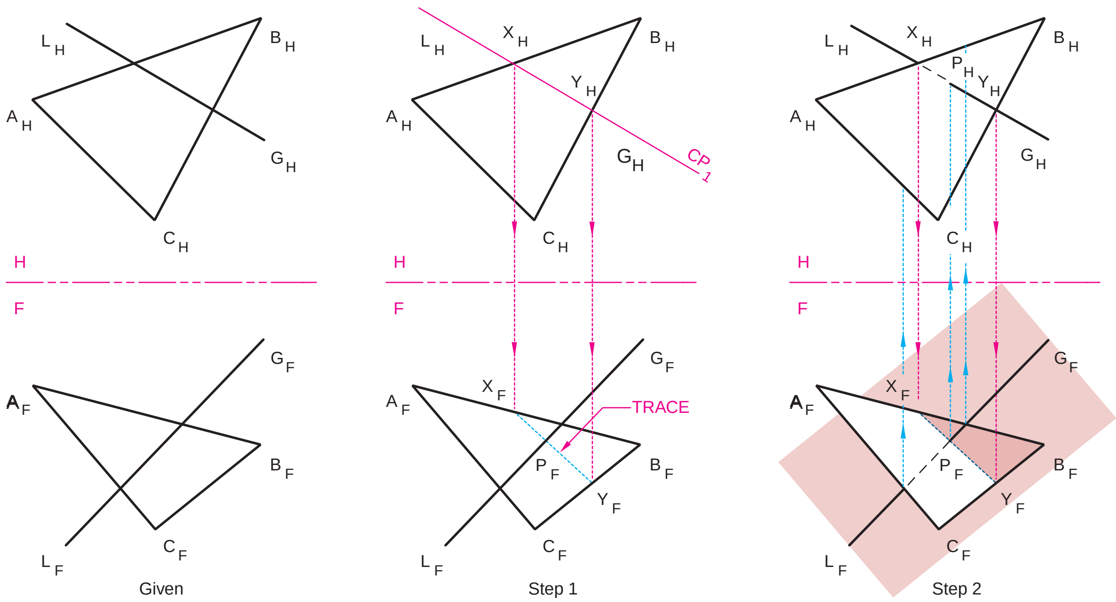


Figure 15.9

Determining the piercing point between a line and a plane, cutting plane method

Determining the Intersection Between a Line and a Plane: Cutting Plane Method

Step 1. Cutting plane CP_1 is placed such that it contains line LG and intersects plane ABC in the top view. The cutting plane cuts through plane ABC at points X and Y, which are then projected into the front view. Point X falls on line AB, and Point Y falls on line BC. Line XY in the front view is called the trace. It shows where the cutting plane cuts the given plane ABC in that view. The intersection of line LG with the trace line in the front view defines the intersection of line LG and plane ABC, at point P.

Step 2. Point P can then be projected back into the top view to define the intersection between line LG and plane ABC in that view. Visibility is determined by projecting intersecting points to adjacent views.

Historical Highlight
Thomas Ewing French (1871–1944)

Like many other notable men, Thomas E. French had many interests: etching, bookplates, paper making, traveling, and many more. The most important of his interests was probably engineering drawing. He was both a teacher and a writer in this subject area. In fact, he even was influential in coining the term "engineering drawing."

French was born and raised in Ohio. There he attended public school, Cooper Academy, and Miami Business College. It was during high school that he also took a night class in mechanical drawing and qualified as a draftsman. He was hired at the Smith-Vale Company and quickly advanced to chief draftsman. However, French decided he was meant for a slightly different path. While working for Smith-Vale he had also taught a drawing class at the YMCA, and this helped lead to his decision to leave his job and attend Ohio State University.

French did well at Ohio State, assisting one of his professors with a book on mechanics, and upon graduating he was immediately hired as a professor. When the university created a Department of Engineering Drawing, French was appointed department head at age 35. French went on to write and collaborate on many textbooks, the first of which was called *A Manual of Engineering Drawing* and was written for the newly created McGraw-Hill Book Company. Another book that he helped to write was *Mechanical Drawing for High Schools,* which was written to help standardize the teaching of engineering drawing in the public school system. These were just two of many books that he wrote, and all of his books were very widely used.

(Courtesy of Ohio State University Photo Archives.)

Thomas E. French was an influential teacher, author, and man. His students found him to be a wonderful teacher, and his books were used everywhere. After his death his family created a fellowship for the Engineering Graphics Department at Ohio State University. The fellowship is used to support graduate students working in engineering graphics. The lead author of this text, Gary Bertoline, was a recipient of this fellowship.

Figure 15.10 illustrates the same procedure with the cutting plane placed first in the front view and intersection points X and Y then projected into the top view to locate the piercing point P.

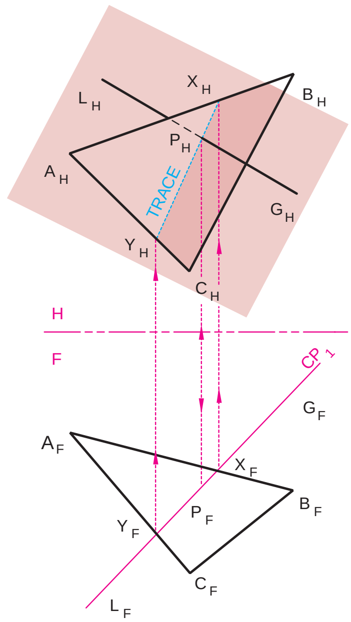
15.2.4 Intersection of Two Planes

The intersection of two planes is a straight line all of whose points are common to both planes. *The line of intersection between two planes is determined by locating the piercing points of lines from one plane with the other plane and drawing a line between the points.* The piercing points are located using either the edge view or cutting plane method.

Edge View Method To determine the line of intersection between two planes using the edge view method, construct an auxiliary view such that one of the planes appears as an edge view. *The piercing points of lines from the other*

Figure 15.10

Determining the piercing point between a line and a plane, cutting plane in the front view



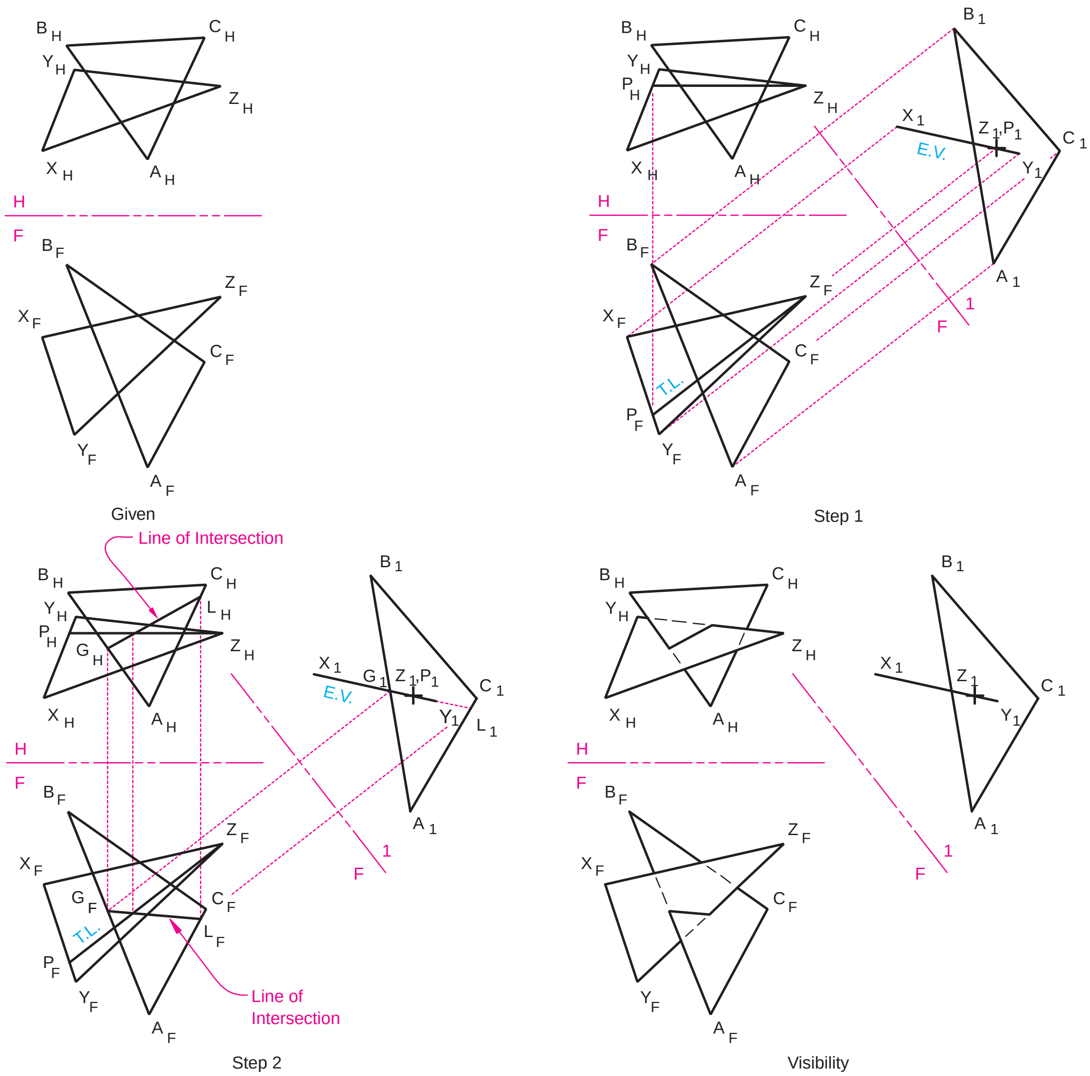


Figure 15.11

Determining the intersection between two planes, edge view method

plane will lie along this edge view. The points of intersection are then projected back into the original views, revealing the line of intersection between the planes.

In Figure 15.11, the top and front views of planes ABC and XYZ are given, and the line of intersection between the planes is to be determined.

Determining the Intersection Between Two Planes: Edge View Method

Step 1. An edge view of plane XYZ is created by constructing an auxiliary view.

Step 2. The edge view of XYZ intersects plane ABC at points G and L. These points are projected back into the front and top views, and a line is drawn between the points; this line is the line of intersection between the planes.

Cutting Plane Method In Figure 15.12 on the next page, the top and front views of planes ABC and XYZ are given, and the line of intersection between the planes is to be determined using cutting planes.

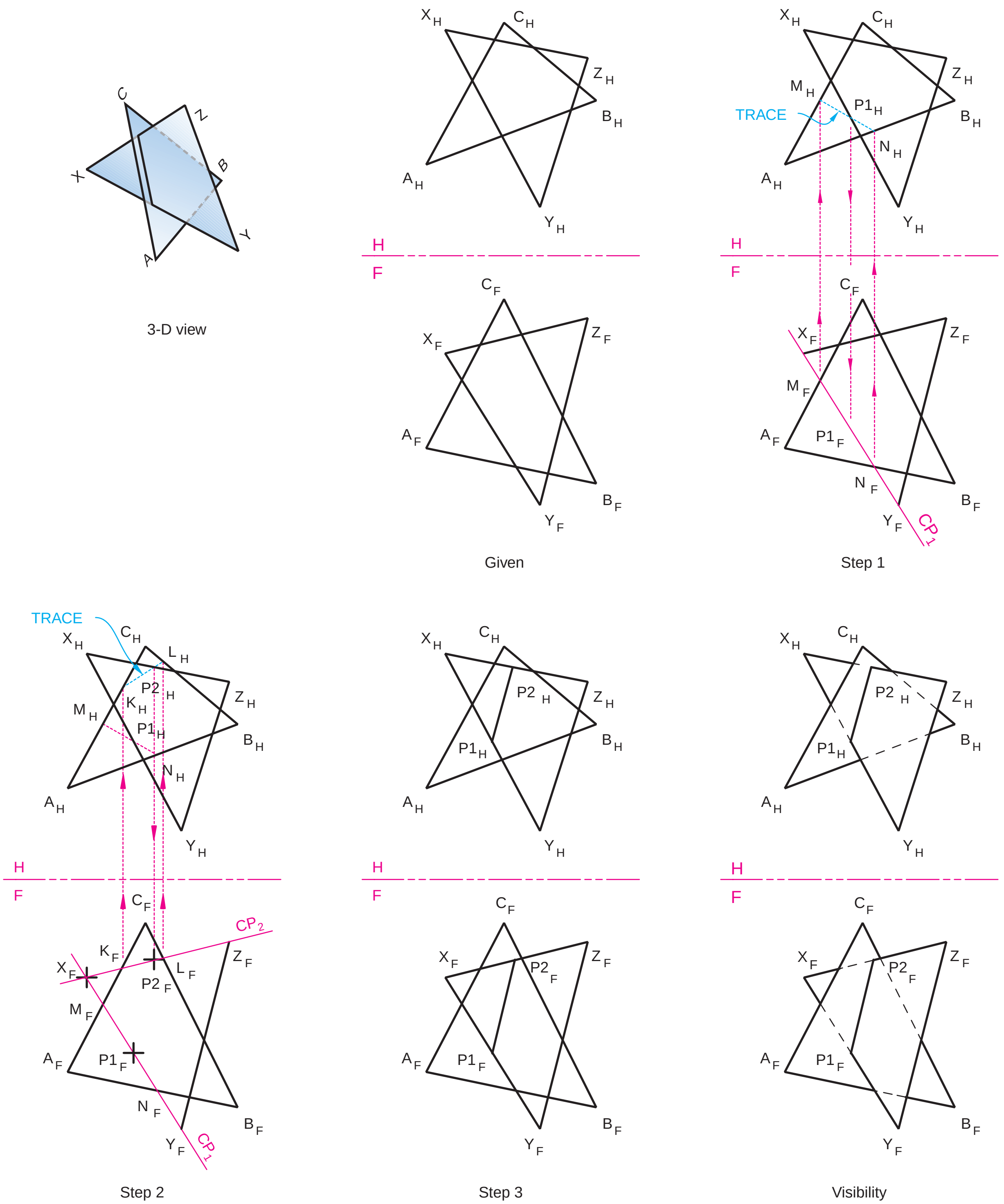


Figure 15.12

Determining the intersection between two planes, cutting plane method

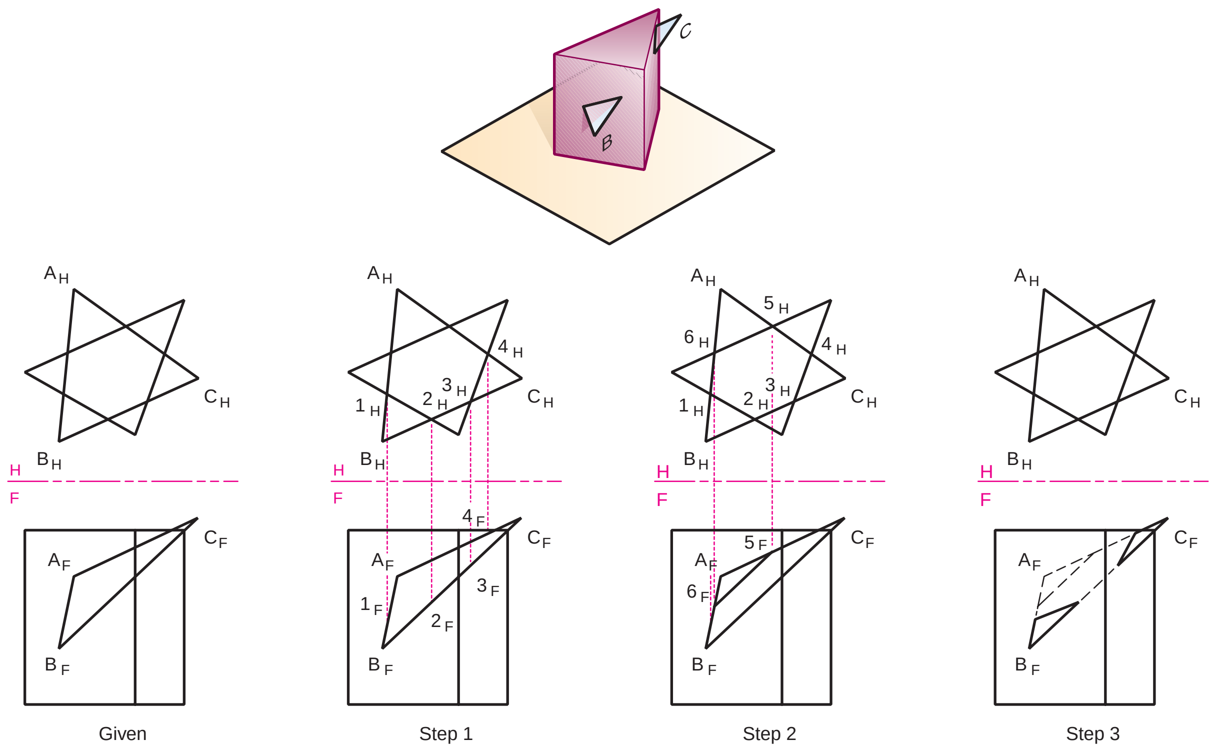


Figure 15.13

Determining the intersection between a plane and a prism

Determining the Intersection Between Two Planes: Cutting Plane Method

Step 1. Create a cutting plane CP_1 through line XY in the front view. This cutting plane intersects plane ABC at points M and N. Project points M and N into the top view and mark these points on lines AC and AB. Draw trace line MN in the top view, and mark where this trace line crosses line XY. Label this as P_1, for the first piercing point. Project P_1 back into the front view, and mark that point P_1.

Step 2. Create the second cutting plane CP_2 in the front view, through line XZ and intersecting plane ABC at points K and L. Project points K and L into the top view on lines AC and BC, draw a trace line between the points, and mark P_2 where this trace line crosses line XZ. Project P_2 back into the front view, and mark that point P_2.

Step 3. Draw a line in both views between piercing points P_1 and P_2; this is the line of intersection between the two planes. The visibility of the two planes in each view is determined using techniques described earlier in this chapter.

15.2.5 Intersection of a Plane and a Solid

Since solids are comprised of plane surfaces, the intersection between a plane and a solid (or between two solids) is determined using the techniques described earlier for finding the intersection between two planes. *To find the intersection between a plane and a solid, determine the piercing points using either cutting planes or auxiliary views; then draw the lines of intersection.*

A Plane and a Prism In Figure 15.13, a solid triangular prism is intersected by triangular plane ABC.

Determining the Intersection Between a Plane and a Prism

Step 1. In the top view, the plane intersects the prism at points 1, 2, 3, and 4. The intersection points are easily established in the top view because the faces of the prism are shown on edge in that view. As stated earlier, points of intersection between planes are located in any view where one of the planes is shown as an edge view.

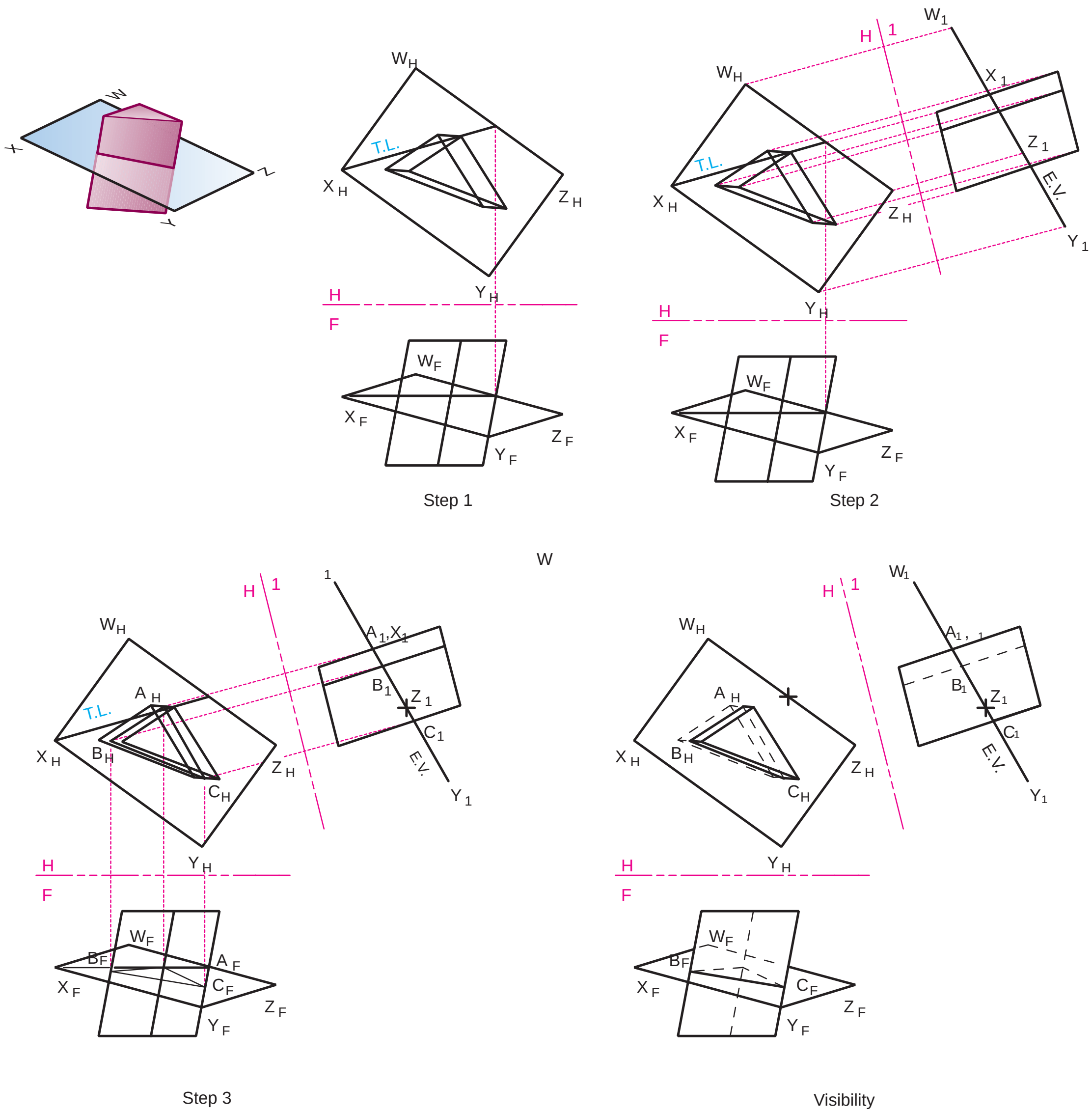


Figure 15.14

Determining the intersection between an oblique plane and a prism using an auxiliary view

These points are projected into the front view, giving the piercing points 1, 2, 3, and 4. The piercing points are connected, revealing the lines of intersection on the surface of the prism.

Step 2. Piercing points 5 and 6 are established in the top view. These points are projected into the front view. The piercing points are connected, revealing the hidden lines of intersection in the front view.

Step 3. Visibility is then determined to complete the front view.

An Oblique Plane and a Prism

When an oblique plane intersects a prism, an auxiliary view can be created to show an edge view of the oblique plane. The points of intersection between the plane and the prism are projected back into the principal views (Figure 15.14). Visibility is determined, then the lines of intersection are constructed.

An Oblique Plane and a Cylinder

The line of intersection between an oblique plane and a cylinder is an ellipse, which can be formed by creating an auxiliary view that shows

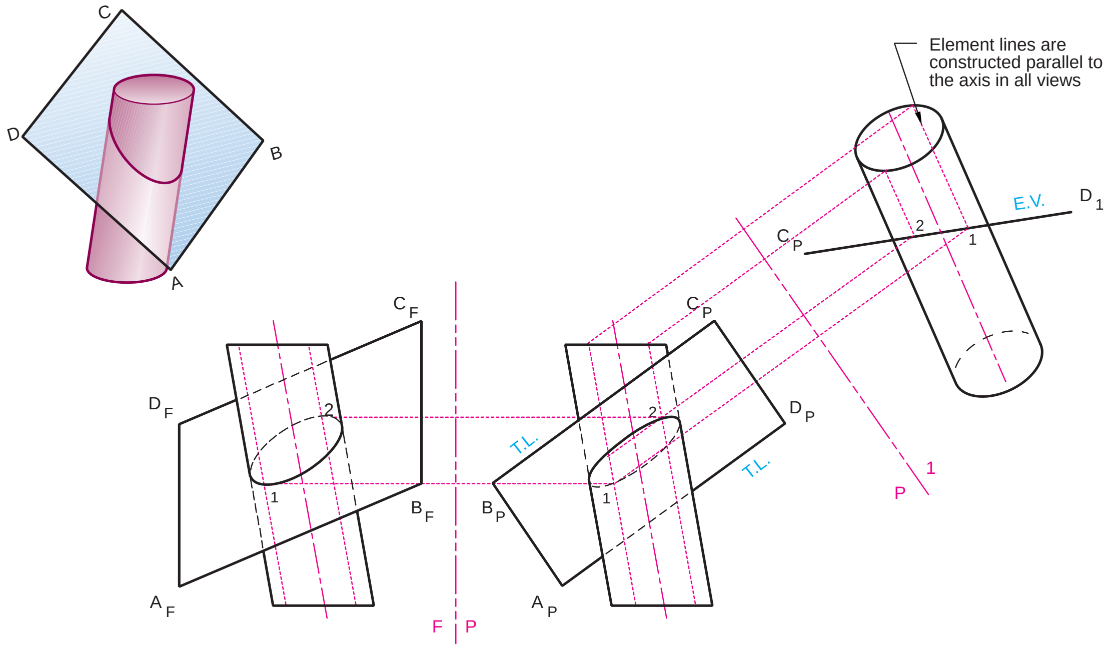


Figure 15.15

Determining the intersection between an oblique plane and a cylinder

the plane as an edge view (Figure 15.15). Element lines are then constructed on the surface of the cylinder, in all the views. In the auxiliary view, the intersection points between the elements and the edge view of the oblique plane are marked and then projected back to the corresponding elements in the principal views. The resulting points are on the elliptical line of intersection between the cylinder and the oblique plane and are connected to create the ellipse.

A Plane and a Cone When a plane intersects a cone and does not intersect the base nor is perpendicular to the axis, the points of intersection form an ellipse. The intersection can be found by passing multiple cutting planes through the cone, either parallel or perpendicular to the cone's axis. Figure 15.16 demonstrates the parallel approach, using a right circular cone. Multiple cutting planes are passed parallel to the axis of the cone and through the apex in the top view. This is done by dividing the base circle of the cone into an equal number of parts, such as 12, as shown in the figure. Elements are drawn on the surface of the cone where the cutting planes intersect that surface in the front view. The piercing points of the elements and the plane are intersections between the plane and the cone. These piercing points are projected to the corresponding elements in the top view to locate the piercing points in that view. These piercing points are

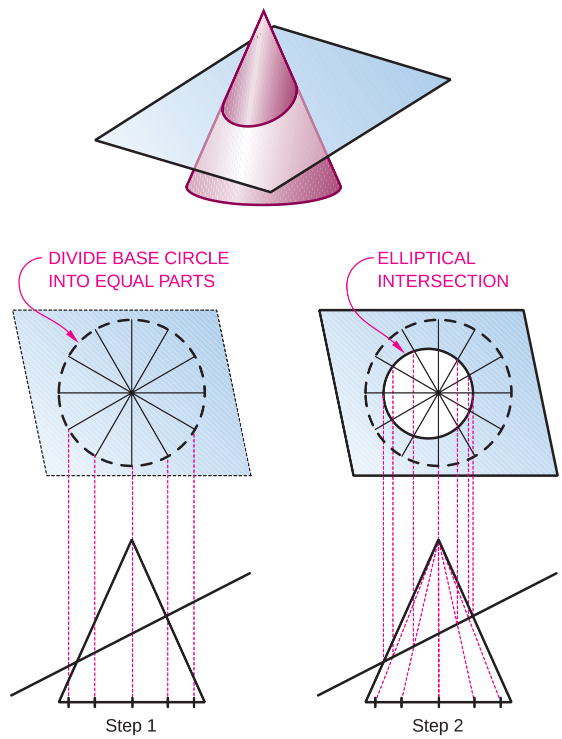


Figure 15.16

Determining the intersection between a plane and a cone

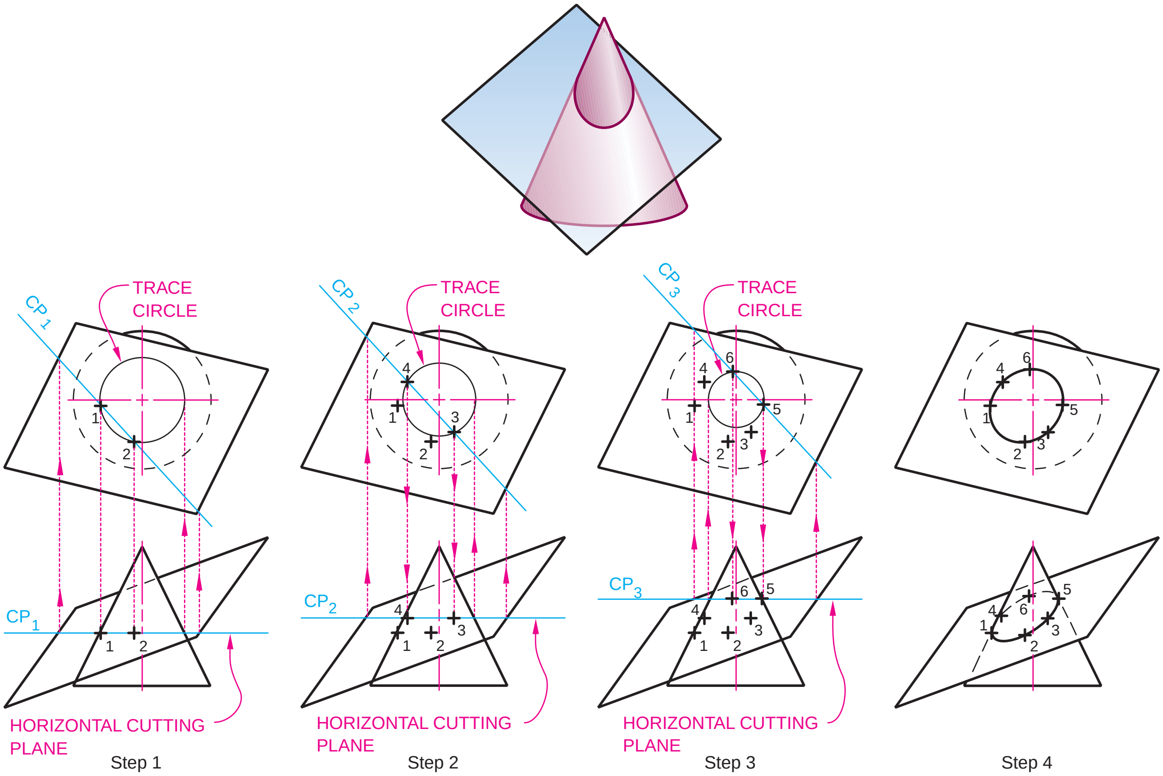


Figure 15.17

Determining the intersection between an oblique plane and a cone

then connected with a smooth curve, forming the elliptical intersection between the cone and the plane.

An Oblique Plane and a Cone Multiple cutting planes can also be used to determine the intersection between an oblique plane and a right circular cone (Figure 15.17).

Determining the Intersection Between an Oblique Plane and a Cone

Step 1. A horizontal cutting plane CP_1 passes through the front view of the cone. This creates a circular view of the cone in the top view and a line on the oblique plane. The first two piercing points 1 and 2 are located at the intersections between line CP_1 and the circle, in the top view. These points are projected back into the front view, to the horizontal cutting plane.

Step 2. A second horizontal cutting plane CP_2 is constructed in the front view, and the process described in the previous step is repeated, creating intersection points 3 and 4, which are also projected back into the front view to the horizontal cutting plane.

Step 3. A third horizontal cutting plane CP_3 is constructed in the front view, as before, creating intersection points 5 and 6, and so on.

Step 4. Additional cutting planes are constructed as necessary until a sufficient number of intersection points are located. The elliptical line of intersection is a smooth curve connecting the piercing points in both the front and top views. Visibility is then determined to complete the construction.

15.2.6 Intersection of Two Solids

The same techniques used to find the intersection between a plane and a solid are used to determine the line of intersection between two solids. Both the edge view method and the cutting plane method can be used.

Two Prisms The intersection between two prisms is constructed by developing an auxiliary view in which the

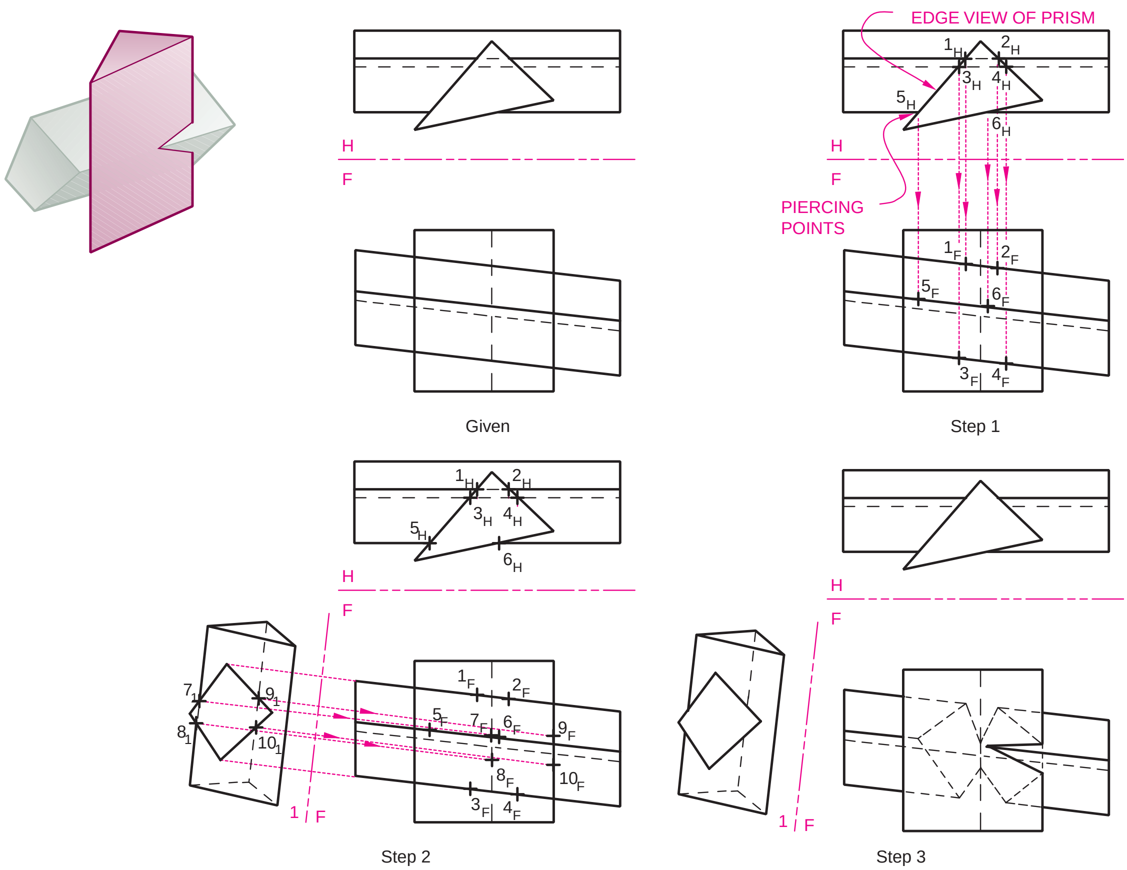


Figure 15.18

Determining the intersection between two prisms

faces of one of the prisms are shown as edges. Piercing points between the edges and the projection of the second prism are located and are projected back into the principal views.

Determining the Intersection Between Two Prisms

Step 1. In Figure 15.18, the faces of one of the prisms are edges in the top view. Piercing points are located where the corners of the second prism intersect the edge views of the first prism. Points 1, 2, 3, 4, 5, and 6 are located and are then projected into the front view.

Step 2. Other piercing points 7, 8, 9, and 10 are located by creating an auxiliary view that shows the other prism's faces on edge, and these points are then projected into the front view.

Step 3. Visibility is determined, then the lines of intersection are drawn in the front view.

A Prism and a Pyramid Straight lines of intersection are created when a prism intersects a pyramid. A combination of the cutting plane and edge view methods can be used to locate this line of intersection (Figure 15.19).

Determining the Intersection Between a Prism and a Pyramid

Step 1. The edge view of the surfaces of the prism is constructed in an auxiliary view by projecting parallel to the edges of the prism that are true length in the front view. The first piercing point P is located in the auxiliary view where pyramid edge OR intersects the edge view of prism side 1–2. Point P is then projected back into the front and top views.

Step 2. In the auxiliary view, a cutting plane is constructed from the apex of the pyramid through the corners of the prism to the base of the pyramid. These lines are

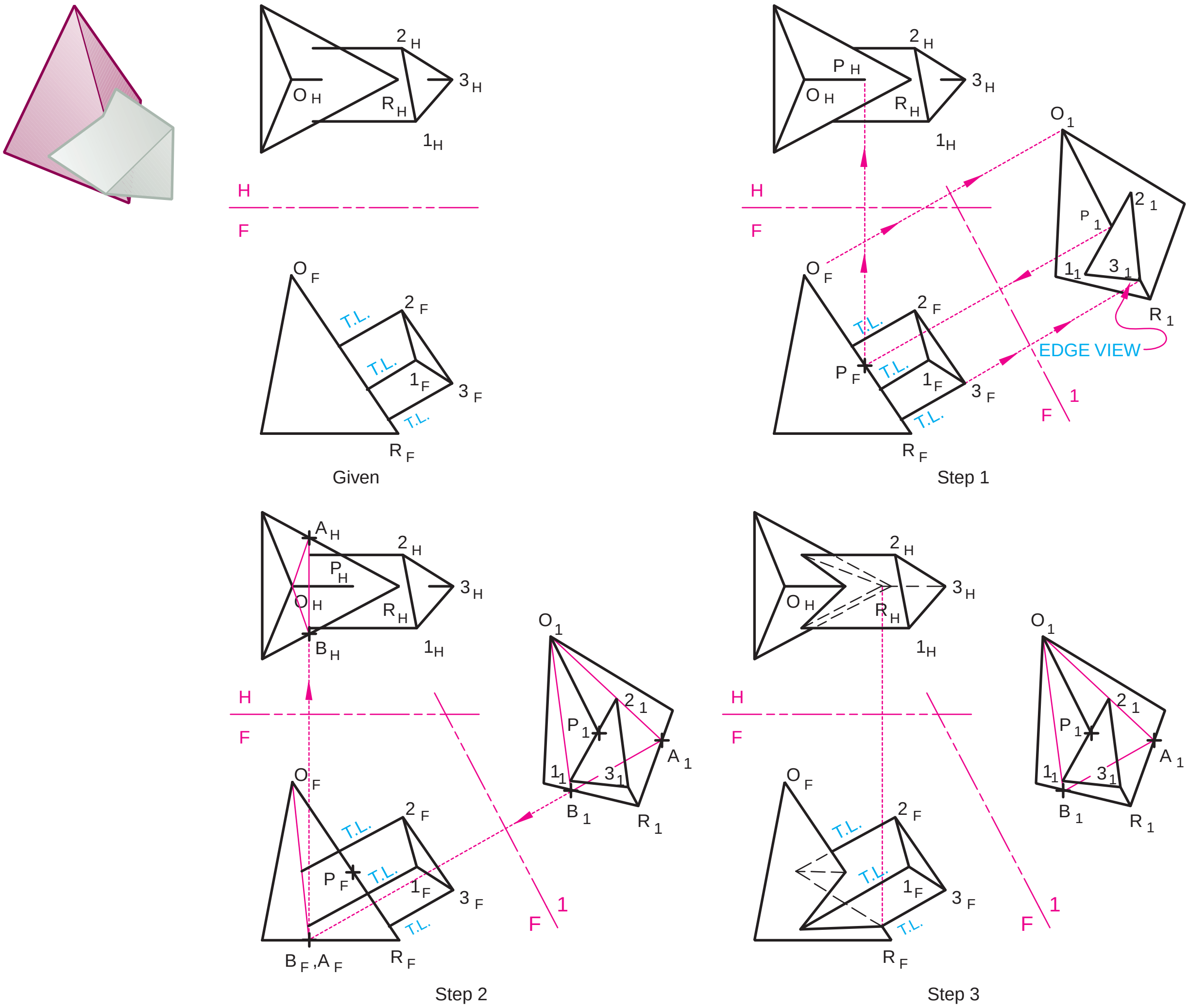


Figure 15.19

Determining the line of intersection between a prism and a pyramid

labeled OA and OB, then projected into the front and top views. The edges of the prism are extended in the front view to intersect the newly constructed lines, which locates the piercing points between the prism and the pyramid.

Step 3. After all the piercing points are located and the visibility is determined, the lines of intersection are drawn in the top and front views.

A Prism and a Cone The intersection between a cone and a prism is a curved line, which is determined using an auxiliary view and cutting planes.

Determining the Intersection Between a Prism and a Cone

Step 1. Figure 15.20 shows an auxiliary view in which the prism faces that intersect the cone are on edge, forming triangle 1–2–3. In the top view, cutting planes are constructed through the apex O of the cone to the base, forming elements on the cone. These elements are projected into the front and auxiliary views and labeled.

Step 2. In the auxiliary view, the intersections between the element lines and one edge view of the prism are marked and then projected back into the front and top views. The intersections between these projections and

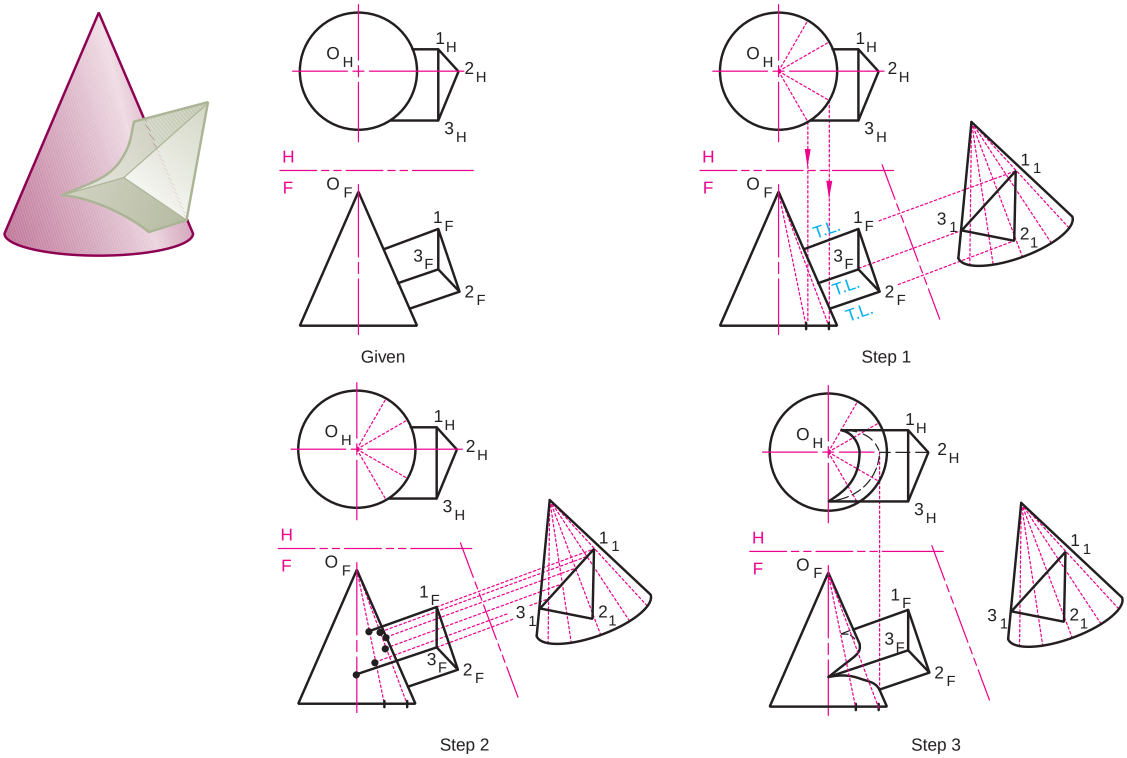


Figure 15.20

Determining the intersection between a prism and a cone

the element lines lie along the curved line of intersection between the cone and the prism in the front and top views.

Step 3. The piercing points for the other edge views of the prism are projected from the auxiliary view back into the front and top views. The correct visibility is determined, and then the piercing points are connected, creating the curved line of intersection in the front and top views.

A Prism and a Cylinder Figure 15.21 on page 732 shows a cylinder intersecting a prism. To determine the line of intersection, use either an auxiliary view showing the intersecting sides of the prism on edge, or use cutting planes. Figure 15.21 uses the auxiliary view approach.

Determining the Intersection Between a Prism and a Cylinder

Step 1. The auxiliary view is created such that the prism is on edge and appears as triangle ABC. Cutting planes are constructed in the top view and labeled CP_1, CP_2, CP_3, CP_4, etc. Locate these cutting planes in the auxiliary view by transferring distances, using dividers.

Step 2. Label all the points where the cutting planes intersect the cylinder in the top view and where the cutting planes intersect the edges of the prism in the auxiliary view. Project both sets of points into the front view.

Step 3. The intersections of the corresponding projections (e.g., the projections from points A: both views) are on the line of intersection between the prism and the cylinder. Determine the correct visibility; then draw the curved line of intersection.

A Cylinder and a Cone The intersection between a cylinder and a cone is a curved line, as shown in Figure 15.22 on page 732. Cutting planes are used to locate intersection points.

Determining the Intersection Between a Cylinder and a Cone

Step 1. For this example, five cutting planes are constructed in the top view, parallel to the axis of the cone and through the apex. These cutting planes form

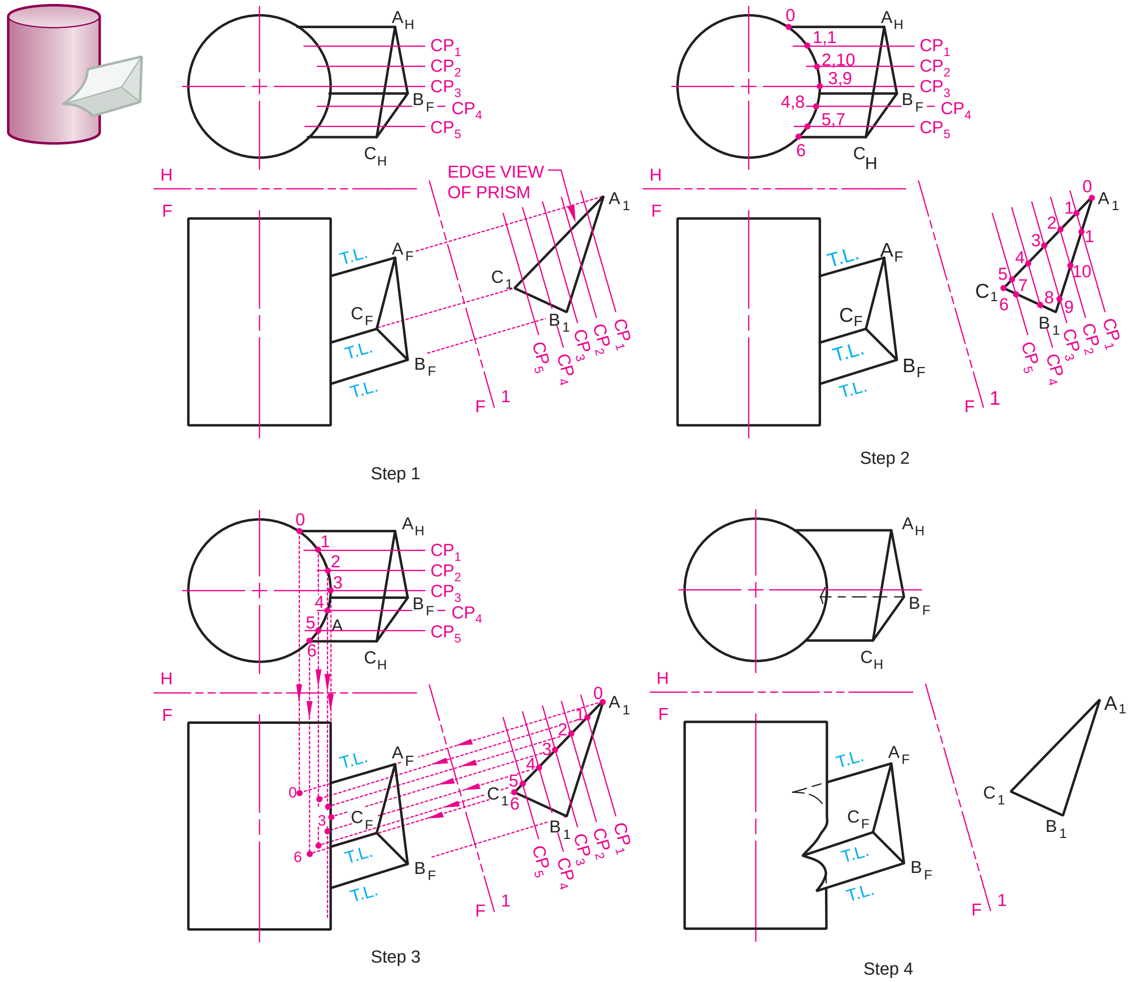


Figure 15.21

Determining the intersection between a prism and a cylinder

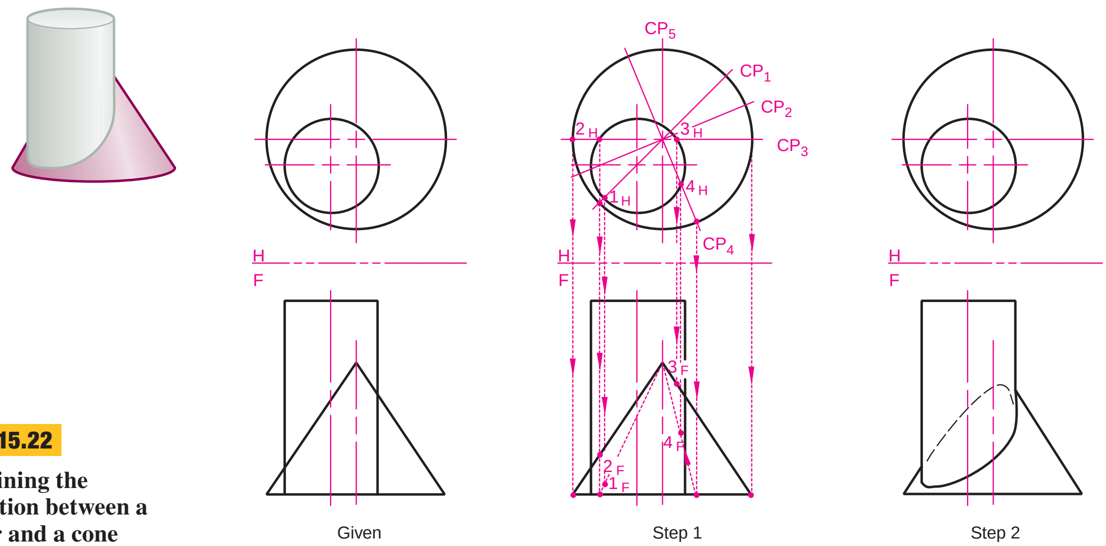


Figure 15.22

Determining the intersection between a cylinder and a cone

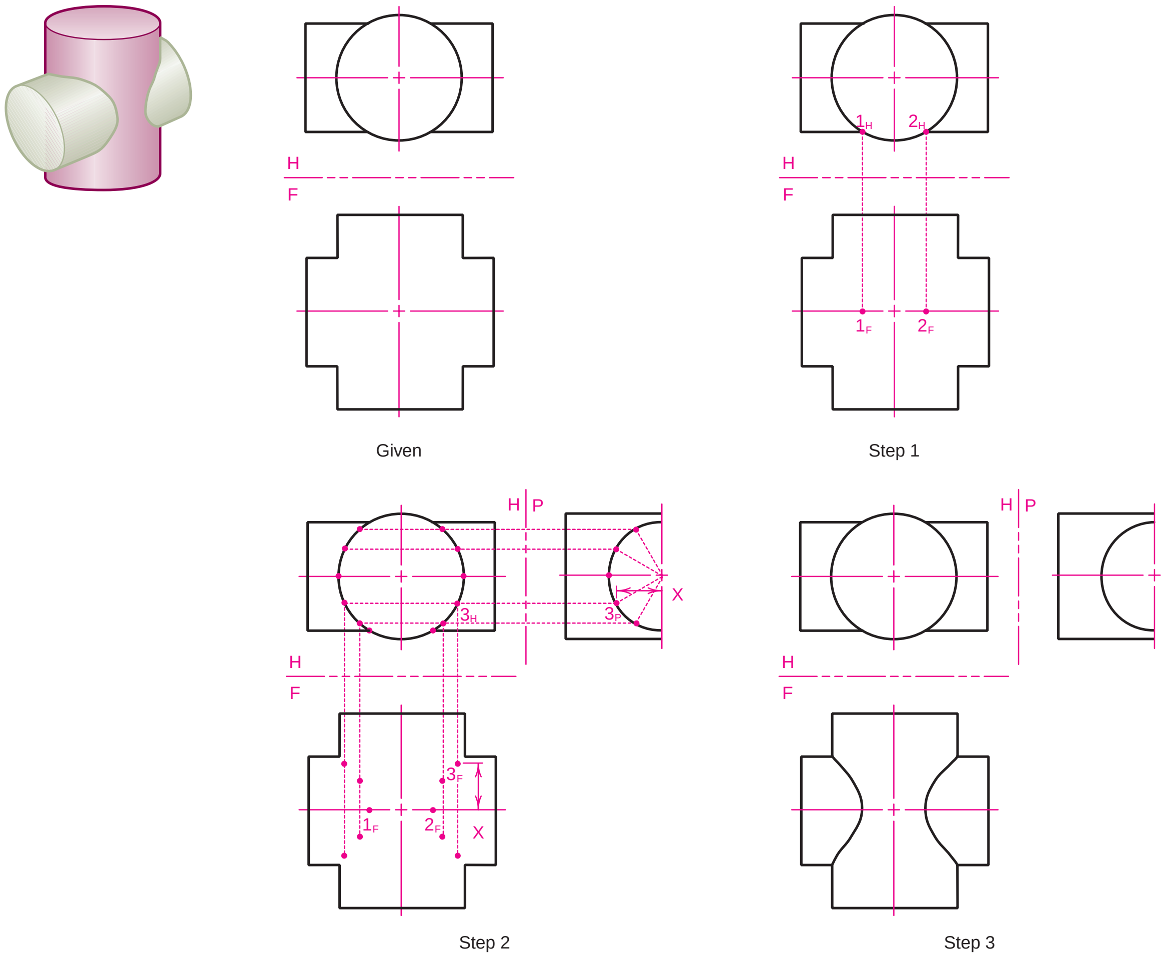


Figure 15.23

Determining the intersection between two perpendicular cylinders

elements on the surface of the cone, which are projected into the front view. In the top view, points are located where each cutting plane intersects the edge of the cylinder. These points are projected into the front view to the corresponding element lines. Additional cutting planes will produce a more accurate representation of the line of intersection.

Step 2. After a sufficient number of points are located in the front view, visibility is determined, and the curved line of intersection is drawn.

Two Perpendicular Cylinders Figure 15.23 shows two intersecting perpendicular cylinders. The line of intersection is a curve. To determine the curved line of intersection, use cutting planes to locate points along the curve.

Determining the Intersection Between Two Perpendicular Cylinders

Step 1. The first points of intersection 1 and 2 are located in the top view, where the outside edge of the smaller cylinder intersects the larger cylinder. These points are projected into the front view and are located along the horizontal center line.

Step 2. A side view adjacent to the top view is constructed. Then, horizontal cutting planes are constructed in the top view and projected into the side view. The cutting plane intersections with the large cylinder are points along the curved line of intersection. The points of intersection are projected into the front view and located using measurements taken from the side view. For example, point 3 is projected into the front view and is located by first