ELEMENTS OF METRIC GEAR TECHNOLOGY

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ELEMENTS OF METRIC GEAR TECHNOLOGY

Gears are some of the most important elements used in machinery. There are few mechanical devices that do not have the need to transmit power and motion between rotating shafts. Gears not only do this most satisfactorily, but can do so with uniform motion and reliability. In addition, they span the entire range of applications from large to small. To summarize:

1. Gears offer positive transmission of power.
2. Gears range in size from small miniature instrument installations, that measure in only several millimeters in diameter, to huge powerful gears in turbine drives that are several meters in diameter.
3. Gears can provide position transmission with very high angular or linear accuracy; such as used in servomechanisms and military equipment.
4. Gears can couple power and motion between shafts whose axes are parallel, intersecting or skew.
5. Gear designs are standardized in accordance with size and shape which provides for widespread interchangeability.

This technical manual is written as an aid for the designer who is a beginner or only superficially knowledgeable about gearing. It provides fundamental theoretical and practical information. Admittedly, it is not intended for experts.

Those who wish to obtain further information and special details should refer to the reference list at the end of this text and other literature on mechanical machinery and components.

SECTION 1 INTRODUCTION TO METRIC GEARS

This technical section is dedicated to details of metric gearing because of its increasing importance. Currently, much gearing in the United States is still based upon the inch system. However, with most of the world metricated, the use of metric gearing in the United States is definitely on the increase, and inevitably at some future date it will be the exclusive system.

It should be appreciated that in the United States there is a growing amount of metric gearing due to increasing machinery and other equipment imports. This is particularly true of manufacturing equipment, such as printing presses, paper machines and machine tools. Automobiles are another major example, and one that impacts tens of millions of individuals. Further spread of metric gearing is inevitable since the world that surrounds the United States is rapidly approaching complete conformance. England and Canada, once bastions of the inch system, are well down the road of metrication, leaving the United States as the only significant exception.

Thus, it becomes prudent for engineers and designers to not only become familiar with metric gears, but also to incorporate them in their designs. Certainly, for export products it is imperative; and for domestic products it is a serious consideration. The U.S. Government, and in particular the military, is increasingly insisting upon metric based equipment designs.

Recognizing that most engineers and designers have been reared in an environment of heavy use of the inch system and that the amount of literature about metric gears is limited, we are offering this technical gear section as an aid to understanding and use of metric gears. In the following pages, metric gear standards are introduced along with information about interchangeability and noninterchangeability. Although gear theory is the same for both the inch and metric systems, the formulas for metric gearing take on a different set of symbols. These equations are fully defined in the metric system. The coverage is thorough and complete with the intention that this be a source for all information about gearing with definition in a metric format.

1.1 Comparison Of Metric Gears With American Inch Gears

1.1.1 Comparison Of Basic Racks

In all modern gear systems, the rack is the basis for tooth design and manufacturing tooling. Thus, the similarities and differences between the two systems can be put into proper perspective with comparison of the metric and inch basic racks.

In both systems, the basic rack is normalized for a unit size. For the metric rack it is 1 module, and for the inch rack it is 1 diametral pitch.

1.1.2 Metric ISO Basic Rack

The standard ISO metric rack is detailed in **Figure 1-1**. It is now the accepted standard for the international community, it having eliminated a number of minor differences that existed between the earlier versions of Japanese, German and Russian modules. For comparison, the standard inch rack is detailed in **Figure 1-2**. Note that there are many similarities. The principal factors are the same for both racks. Both are normalized for unity; that is, the metric rack is specified in terms of 1 module, and the inch rack in terms of 1 diametral pitch.

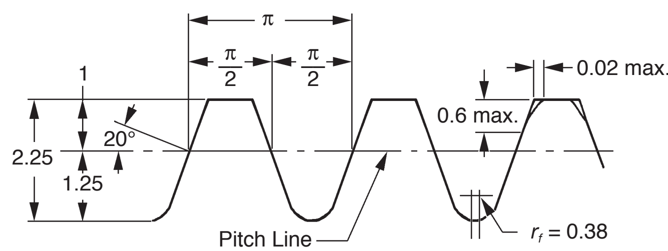
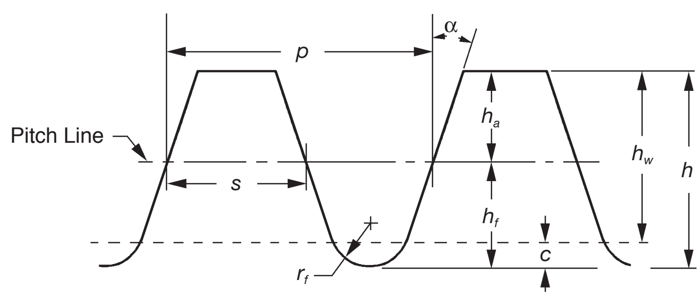


Fig. 1-1 The Basic Metric Rack From ISO 53 Normalized for Module 1



h_a = Addendum	h_w = Working Depth	r_f = Root Radius
h_f = Dedendum	h = Whole Depth	s = Circular Tooth Thickness
c = Clearance	p = Circular Pitch	α = Pressure Angle

Fig. 1-2 The Basic Inch Diametral Pitch Rack Normalized for 1 Diametral Pitch

From the normalized metric rack, corresponding dimensions for any module are obtained by multiplying each rack dimension by the value of the specific module m. The major tooth parameters are defined by the standard, as:

Tooth Form: Straight-sided full depth, forming the basis of a family of full depth interchangeable gears.

Pressure Angle: A 20° pressure angle, which conforms to worldwide acceptance of this as the most versatile pressure angle.

Addendum: This is equal to the module m, which is similar to the inch value that becomes $1/p$.

Dedendum: This is 1.25 m; again similar to the inch rack value.

Root Radius: The metric rack value is slightly greater than the American inch rack value.

Tip Radius: A maximum value is specified. This is a deviation from the American inch rack which does not specify a rounding.

1.1.3 Comparison Of Gear Calculation Equations

Most gear equations that are used for diametral pitch inch gears are equally applicable to metric gears if the module m is substituted for diametral pitch. However, there are exceptions when it is necessary to use dedicated metric equations. Thus, to avoid confusion and errors, it is most effective to work entirely with and within the metric system.

1.2 Metric Standards Worldwide

1.2.1 ISO Standards

Metric standards have been coordinated and standardized by the International Standards Organization (ISO). A listing of the most pertinent standards is given in **Table 1-1**.

1.2.2 Foreign Metric Standards

Most major industrialized countries have been using metric gears for a long time and consequently had developed their own standards prior to the establishment of ISO and SI units. In general, they are very similar to the ISO standards. The key foreign metric standards are listed in **Table 1-2** for reference.

1.3 Japanese Metric Standards In This Text

1.3.1 Application Of JIS Standards

Japanese Industrial Standards (JIS) define numerous engineering subjects including gearing. The originals are generated in Japanese, but they are translated and published in English by the Japanese Standards Association.

Considering that many metric gears are produced in Japan, the JIS standards may apply. These essentially conform to all aspects of the ISO standards.

Table 1-1 ISO Metric Gearing Standards

ISO 53:1974	Cylindrical gears for general and heavy engineering – Basic rack
ISO 54:1977	Cylindrical gears for general and heavy engineering – Modules and diametral pitches
ISO 677:1976	Straight bevel gears for general and heavy engineering – Basic rack
ISO 678:1976	Straight bevel gears for general and heavy engineering – Modules and diametral pitches
ISO 701:1976	International gear notation – symbols for geometrical data
ISO 1122-1:1983	Glossary of gear terms – Part 1: Geometrical definitions
ISO 1328:1975	Parallel involute gears – ISO system of accuracy
ISO 1340:1976	Cylindrical gears – Information to be given to the manufacturer by the purchaser in order to obtain the gear required
ISO 1341:1976	Straight bevel gears – Information to be given to the manufacturer by the purchaser in order to obtain the gear required
ISO 2203:1973	Technical drawings – Conventional representation of gears
ISO 2490:1975	Single-start solid (monobloc) gear hobs with axial keyway, 1 to 20 module and 1 to 20 diametral pitch – Nominal dimensions
ISO/TR 4467:1982	Addendum modification of the teeth of cylindrical gears for speed-reducing and speed-increasing gear pairs
ISO 4468:1982	Gear hobs – Single-start – Accuracy requirements
ISO 8579-1:1993	Acceptance code for gears – Part 1: Determination of airborne sound power levels emitted by gear units
ISO 8579-2:1993	Acceptance code for gears – Part 2: Determination of mechanical vibrations of gear units during acceptance testing
ISO/TR 10064-1:1992	Cylindrical gears – Code of inspection practice – Part 1: Inspection of corresponding flanks of gear teeth

Table 1-2 Foreign Metric Gear Standards

AUSTRALIA		
AS B 62	1965	Bevel gears
AS B 66	1969	Worm gears (inch series)
AS B 214	1966	Geometrical dimensions for worm gears – Units
AS B 217	1966	Glossary for gearing
AS 1637		International gear notation symbols for geometric data (similar to ISO 701)

FRANCE		
NF E 23-001	1972	Glossary of gears (similar to ISO 1122)
NF E 23-002	1972	Glossary of worm gears
NF E 23-005	1965	Gearing – Symbols (similar to ISO 701)
NF E 23-006	1967	Tolerances for spur gears with involute teeth (similar to ISO 1328)
NF E 23-011	1972	Cylindrical gears for general and heavy engineering – Basic rack and modules (similar to ISO 467 and ISO 53)
NF E 23-012	1972	Cylindrical gears – Information to be given to the manufacturer by the producer
NF L 32-611	1955	Calculating spur gears to NF L 32-610

Continued on following page

Table 1-2 (Cont.) Foreign Metric Gear Standards

GERMANY – DIN (Deutsches Institut für Normung)		
DIN 37	12.61	Conventional and simplified representation of gears and gear pairs [4]
DIN 780 Pt 1	05.77	Series of modules for gears – Modules for spur gears [4]
DIN 780 Pt 2	05.77	Series of modules for gears – Modules for cylindrical worm gear transmissions [4]
DIN 867	02.86	Basic rack tooth profiles for involute teeth of cylindrical gears for general and heavy engineering [5]
DIN 868	12.76	General definitions and specification factors for gears, gear pairs and gear trains [11]
DIN 3961	08.78	Tolerances for cylindrical gear teeth – Bases [8]
DIN 3962 Pt 1	08.78	Tolerances for cylindrical gear teeth – Tolerances for deviations of individual parameters [11]
DIN 3962 Pt 2	08.78	Tolerances for cylindrical gear teeth – Tolerances for tooth trace deviations [4]
DIN 3962 Pt 3	08.78	Tolerances for cylindrical gear teeth – Tolerances for pitch-span deviations [4]
DIN 3963	08.78	Tolerances for cylindrical gear teeth – Tolerances for working deviations [11]
DIN 3964	11.80	Deviations of shaft center distances and shaft position tolerances of casings for cylindrical gears [4]
DIN 3965 Pt 1	08.86	Tolerancing of bevel gears – Basic concepts [5]
DIN 3965 Pt 2	08.86	Tolerancing of bevel gears – Tolerances for individual parameters [11]
DIN 3965 Pt 3	08.86	Tolerancing of bevel gears – Tolerances for tangential composite errors [11]
DIN 3965 Pt 4	08.86	Tolerancing of bevel gears – Tolerances for shaft angle errors and axes intersection point deviations [5]
DIN 3966 Pt 1	08.78	Information on gear teeth in drawings – Information on involute teeth for cylindrical gears [7]
DIN 3966 Pt 2	08.78	Information on gear teeth in drawings – Information on straight bevel gear teeth [6]
DIN 3967	08.78	System of gear fits – Backlash, tooth thickness allowances, tooth thickness tolerances – Principles [12]
DIN 3970 Pt 1	11.74	Master gears for checking spur gears – Gear blank and tooth system [8]
DIN 3970 Pt 2	11.74	Master gears for checking spur gears – Receiving arbors [4]
DIN 3971	07.80	Definitions and parameters for bevel gears and bevel gear pairs [12]
DIN 3972	02.52	Reference profiles of gear-cutting tools for involute tooth systems according to DIN 867 [4]
DIN 3975	10.76	Terms and definitions for cylindrical worm gears with shaft angle 90° [9]
DIN 3976	11.80	Cylindrical worms – Dimensions, correlation of shaft center distances and gear ratios of worm gear drives [6]
DIN 3977	02.81	Measuring element diameters for the radial or diametral dimension for testing tooth thickness of cylindrical gears [8]
DIN 3978	08.76	Helix angles for cylindrical gear teeth [5]
DIN 3979	07.79	Tooth damage on gear trains – Designation, characteristics, causes [11]
DIN 3993 Pt 1	08.81	Geometrical design of cylindrical internal involute gear pairs – Basic rules [17]
DIN 3993 Pt 2	08.81	Geometrical design of cylindrical internal involute gear pairs – Diagrams for geometrical limits of internal gear-pinion matings [15]
DIN 3993 Pt 3	08.81	Geometrical design of cylindrical internal involute gear pairs – Diagrams for the determination of addendum modification coefficients [15]
DIN 3993 Pt 4	08.81	Geometrical design of cylindrical internal involute gear pairs – Diagrams for limits of internal gear-pinion type cutter matings [10]
DIN 3998 Suppl 1	09.76	Denominations on gear and gear pairs – Alphabetical index of equivalent terms [10]
DIN 3998 Pt 1	09.76	Denominations on gears and gear pairs – General definitions [11]
DIN 3998 Pt 2	09.76	Denominations on gears and gear pairs – Cylindrical gears and gear pairs [11]
DIN 3998 Pt 3	09.76	Denominations on gears and gear pairs – Bevel and hypoid gears and gear pairs [9]
DIN 3998 Pt 4	09.76	Denominations on gears and gear pairs – Worm gear pairs [8]
DIN 58405 Pt 1	05.72	Spur gear drives for fine mechanics –Scope, definitions, principal design data, classification [7]
DIN 58405 Pt 2	05.72	Spur gear drives for fine mechanics – Gear fit selection, tolerances, allowances [9]
DIN 58405 Pt 3	05.72	Spur gear drives for fine mechanics – Indication in drawings, examples for calculation [12]
DIN 58405 Pt 4	05.72	Spur gear drives for fine mechanics – Tables [15]
DIN ISO 2203	06.76	Technical Drawings – Conventional representation of gears

NOTES:

1. Standards available in English from: ANSI, 1430 Broadway, New York, NY 10018; or Beuth Verlag GmbH, Burggrafenstrasse 6, D-10772 Berlin, Germany; or Global Engineering Documents, Inverness Way East, Englewood, CO 80112-5704
2. *Above data was taken from: DIN Catalogue of Technical Rules 1994, Supplement, Volume 3, Translations*

Table 1-2 (Cont.) Foreign Metric Gear Standards

ITALY		
UNI 3521	1954	Gearing – Module series
UNI 3522	1954	Gearing – Basic rack
UNI 4430	1960	Spur gear – Order information for straight and bevel gear
UNI 4760	1961	Gearing – Glossary and geometrical definitions
UNI 6586	1969	Modules and diametral pitches of cylindrical and straight bevel gears for general and heavy engineering (corresponds to ISO 54 and 678)
UNI 6587	1969	Basic rack of cylindrical gears for standard engineering (corresponds to ISO 53)
UNI 6588	1969	Basic rack of straight bevel gears for general and heavy engineering (corresponds to ISO 677)
UNI 6773	1970	International gear notation – Symbols for geometrical data (corresponds to ISO 701)

Continued on following page

Table 1-2 (Cont.) Foreign Metric Gear Standards

JAPAN – JIS (Japanese Industrial Standards)		
B 0003	1989	Drawing office practice for gears
B 0102	1988	Glossary of gear terms
B 1701	1973	Involute gear tooth profile and dimensions
B 1702	1976	Accuracy for spur and helical gears
B 1703	1976	Backlash for spur and helical gears
B 1704	1978	Accuracy for bevel gears
B 1705	1973	Backlash for bevel gears
B 1721	1973	Shapes and dimensions of spur gears for general engineering
B 1722	1974	Shape and dimensions of helical gears for general use
B 1723	1977	Dimensions of cylindrical worm gears
B 1741	1977	Tooth contact marking of gears
B 1751	1976	Master cylindrical gears
B 1752	1989	Methods of measurement of spur and helical gears
B 1753	1976	Measuring method of noise of gears
B 4350	1991	Gear cutter tooth profile and dimensions
B 4351	1985	Straight bevel gear generating cutters
B 4354	1988	Single thread hobs
B 4355	1988	Single thread fine pitch hobs
B 4356	1985	Pinion type cutters
B 4357	1988	Rotary gear shaving cutters
B 4358	1991	Rack type cutters

NOTE:

Standards available in English from: ANSI, 1430 Broadway, New York, NY 10018; or International Standardization Cooperation Center, Japanese Standards Association, 4-1-24 Akasaka, Minato-ku, Tokyo 107

Table 1-2 (Cont.) Foreign Metric Gear Standards

UNITED KINGDOM – BSI (British Standards Institute)		
BS 235	1972	Specification of gears for electric traction
BS 436 Pt 1	1987	Spur and helical gears – Basic rack form, pitches and accuracy (diametral pitch series)
BS 436 Pt 2	1984	Spur and helical gears – Basic rack form, modules and accuracy (1 to 50 metric module)
BS 436 Pt 3	1986	(Parts 1 & 2 related but not equivalent with ISO 53, 54, 1328, 1340 & 1341) Spur gear and helical gears – Method for calculation of contact and root bending stresses, limitations for metallic involute gears (Related but not equivalent with ISO / DIS 6336 / 1, 2 & 3)
BS 721 Pt 1	1984	Specification for worm gearing – Imperial units
BS 721 Pt 2	1983	Specification for worm gearing – Metric units
BS 978 Pt 1	1984	Specification for fine pitch gears – Involute spur and helical gears
BS 978 Pt 2	1984	Specification for fine pitch gears – Cycloidal type gears
BS 978 Pt 3	1984	Specification for fine pitch gears – Bevel gears
BS 978 Pt 4	1965	Specification for fine pitch gears – Hobs and cutters
BS 1807	1981	Specification for marine propulsion gears and similar drives: metric module
BS 2007	1983	Specification for circular gear shaving cutters, 1 to 8 metric module, accuracy requirements
BS 2062 Pt 1	1985	Specification for gear hobs – Hobs for general purpose: 1 to 20 d.p., inclusive
BS 2062 Pt 2	1985	Specification for gear hobs – Hobs for gears for turbine reduction and similar drives
BS 2518 Pt 1	1983	Specification for rotary form relieved gear cutters – Diametral pitch
BS 2518 Pt 2	1983	Specification for rotary relieved gear cutters – Metric module
BS 2519 Pt 1	1976	Glossary for gears – Geometrical definitions
BS 2519 Pt 2	1976	Glossary for gears – Notation (symbols for geometrical data for use in gear rotation)
BS 2697	1976	Specification for rack type gear cutters
BS 3027	1968	Specification for dimensions of worm gear units
BS 3696 Pt 1	1984	Specification for master gears – Spur and helical gears (metric module)
BS 4517	1984	Dimensions of spur and helical geared motor units (metric series)
BS 4582 Pt 1	1984	Fine pitch gears (metric module) – Involute spur and helical gears
BS 4582 Pt 2	1986	Fine pitch gears (metric module) – Hobs and cutters
BS 5221	1987	Specifications for general purpose, metric module gear hobs
BS 5246	1984	Specifications for pinion type cutters for spur gears – 1 to 8 metric module
BS 6168	1987	Specification for nonmetallic spur gears

NOTE:

Standards available from: ANSI, 1430 Broadway, New York, NY 10018; or BSI, Linford Wood, Milton Keynes MK146LE, United Kingdom

1.3.2 Symbols

Gear parameters are defined by a set of standardized symbols that are defined in JIS B 0121 (1983). These are reproduced in **Table 1-3**.

The JIS symbols are consistent with the equations given in this text and are consistent with JIS standards. Most differ from typical American symbols, which can be confusing to the first time metric user. To assist, **Table 1-4** is offered as a cross list.

Table 1-3A The Linear Dimensions And Circular Dimensions

Terms	Symbols
Center Distance	a
Circular Pitch (General)	p
Standard Circular Pitch	p
Radial Circular Pitch	p_t
Circular Pitch Perpendicular to Tooth	p_n
Axial Pitch	p_x
Normal Pitch	p_b
Radial Normal Pitch	p_{bt}
Normal Pitch Perpendicular to Tooth	p_{bn}
Whole Depth	h
Addendum	h_a
Dedendum	h_f
Caliper Tooth Height	$\bar{h}$
Working Depth	h' h_w
Tooth Thickness (General)	s
Circular Tooth Thickness	s
Base Circle Circular Tooth Thickness	s_b
Chordal Tooth Thickness	$\bar{s}$
Span Measurement	W
Root Width	e
Top Clearance	c
Circular Backlash	j_t
Normal Backlash	j_n
Blank Width	b
Working Face Width	b' b_w

Terms	Symbols
Lead	p_z
Contact Length	g_a
Contact Length of Approach	g_f
Contact Length of Recess	g_α
Contact Length of Overlap	g_α
Diameter (General)	d
Standard Pitch Diameter	d
Working Pitch Diameter	d' d_w
Outside Diameter	d_a
Base Diameter	d_b
Root Diameter	d_f
Radius (General)	r
Standard Pitch Radius	r
Working Pitch Radius	r' r_w
Outside Radius	r_a
Base Radius	r_b
Root Radius	r_f
Radius of Curvature	ρ
Cone Distance (General)	R
Cone Distance	R_e
Mean Cone Distance	R_m
Inner Cone Distance	R_i
Back Cone Distance	R_v
Mounting Distance	*A
Offset Distance	*E

* These terms and symbols are specific to JIS Standard

Table 1-3B Angular Dimensions

Terms	Symbols
Pressure Angle (General)	α
Standard Pressure Angle	α
Working Pressure Angle	α' or α_w
Cutter Pressure Angle	α_0
Radial Pressure Angle	α_t
Pressure Angle Normal to Tooth	α_n
Axial Pressure Angle	α_x
Helix Angle (General)	β
Standard Pitch Cylinder Helix Angle	β
Outside Cylinder Helix Angle	β_a
Base Cylinder Helix Angle	β_b
Lead Angle (General)	γ
Standard Pitch Cylinder Lead Angle	γ
Outside Cylinder Lead Angle	γ_a
Base Cylinder Lead Angle	γ_b

Terms	Symbols
Shaft Angle	Σ
Cone Angle (General)	δ
Pitch Cone Angle	δ
Outside Cone Angle	δ_a
Root Cone Angle	δ_f
Addendum Angle	θ_a
Dedendum Angle	θ_f
Radial Contact Angle	ϕ_a
Overlap Contact Angle	ϕ_β
Overall Contact Angle	ϕ_r
Angular Pitch of Crown Gear	τ
Involute Function	$\mathrm{inv}\alpha$

Table 1-3C Size Numbers, Ratios & Speed Terms

Terms	Symbols
Number of Teeth	z
Equivalent Spur Gear Number of Teeth	z_v
Number of Threads in Worm	z_w
Number of Teeth in Pinion	z_l
Number of Teeth Ratio	u
Speed Ratio	i
Module	m
Radial Module	m_t
Normal Module	m_n
Axial Module	m_x

Terms	Symbols
Contact Ratio	ε
Radial Contact Ratio	ε_α
Overlap Contact Ratio	ε_β
Total Contact Ratio	ε_γ
Specific Slide	*σ
Angular Speed	ω
Linear or Tangential Speed	v
Revolutions per Minute	n
Coefficient of Profile Shift	x
Coefficient of Center Distance Increase	y

Continued on following page

NOTE: The term "Radial" is used to denote parameters in the plane of rotation perpendicular to the axis.

Table 1-3D Accuracy/Error Terms

Terms	Symbols	Terms	Symbols
Single Pitch Error	f_{pt}	Normal Pitch Error	f_{pb}
Pitch Variation	$*f_u$ or f_{pu}	Involute Profile Error	f_f
Partial Accumulating Error (Over Integral k teeth)	F_{pk}	Runout Error	F_r
Total Accumulated Pitch Error	F_p	Lead Error	F_β

* These terms and symbols are specific to JIS Standards

Table 1-4 Equivalence of American and Japanese Symbols

American Symbol	Japanese Symbol	Nomenclature
B	j	backlash, linear measure along pitch circle
B_{LA}	j_t	backlash, linear measure along line-of-action
${}_aB$	j_n	backlash in arc minutes
C	a	center distance
ΔC	Δa	change in center distance
C_o	a_w	operating center distance
C_{std}		standard center distance
D	d	pitch diameter
D_b	d_b	base circle diameter
D_o	d_a	outside diameter
D_R	d_f	root diameter
F	b	face width
K	K	factor, general
L	L	length, general; also lead of worm
M		measurement over-pins
N	z	number of teeth, usually gear
N_c	z_c	critical number of teeth for no undercutting
h_t	h	whole depth
m_p	ε	contact ratio
n	z_1	number of teeth, pinion
n_w	z_w	number of threads in worm
p_a	p_x	axial pitch
p_b	p_b	base pitch
p_c	p	circular pitch
p_{cn}	p_n	normal circular pitch
r	r	pitch radius, pinion
r_b	r_b	base circle radius, pinion
r_f	r_f	fillet radius
r_o	r_a	outside radius, pinion
t	s	tooth thickness, and for general use, for tolerance

American Symbol	Japanese Symbol	Nomenclature
N_v	z_v	virtual number of teeth for helical gear
P_d	p	diametral pitch
P_{dn}	p_n	normal diametral pitch
P_t		horsepower, transmitted
R	r	pitch radius, gear or general use
R_b	r_b	base circle radius, gear
R_o	r_a	outside radius, gear
R_T		testing radius
T	s	tooth thickness, gear
W_b		beam tooth strength
Y		Lewis factor, diametral pitch
Z	i	mesh velocity ratio
a	h_a	addendum
b	h_f	dedendum
c	c	clearance
d	d	pitch diameter, pinion
d_w	d_p	pin diameter, for over-pins measurement
e		eccentricity
h_k	h_w	working depth
y_c		Lewis factor, circular pitch
γ	δ	pitch angle, bevel gear
θ		rotation angle, general
λ	γ	lead angle, worm gearing
μ		mean value
ν		gear stage velocity ratio
ϕ	α	pressure angle
ϕ_o	α_w	operating pressure angle
Ψ	β	helix angle (β_b=base helix angle; β_w = operating helix angle)
ω		angular velocity
$\mathrm{inv}\phi$	$\mathrm{inv}\alpha$	involute function

1.3.3 Terminology

Terms used in metric gearing are identical or are parallel to those used for inch gearing. The one major exception is that metric gears are based upon the module, which for reference may be considered as the inversion of a metric unit diametral pitch.

Terminology will be appropriately introduced and defined throughout the text.

There are some terminology difficulties with a few of the descriptive words used by the Japanese JIS standards when translated into English. One particular example is the Japanese use of the term "radial" to describe measures such as what Americans term circular pitch. This also crops up with contact ratio. What Americans refer to as contact ratio in the plane of rotation, the Japanese equivalent is called "radial contact ratio". This can be both confusing and annoying. Therefore, since this technical section is being used outside Japan, and the American term is more realistically descriptive, in this text we will use the American term "circular" where it is meaningful. However, the applicable Japanese symbol will be used. Other examples of giving preference to the American terminology will be identified where it occurs.

1.3.4 Conversion

For those wishing to ease themselves into working with metric gears by looking at them in terms of familiar inch gearing relationships and mathematics, **Table 1-5** is offered as a means to make a quick comparison.

Table 1-5 Spur Gear Design Formulas

To Obtain	From Known	Use This Formula*
Pitch Diameter	Module	$D = mN$
Circular Pitch	Module	$p_c = m\pi = \frac{D}{N}\pi$
Module	Diametral Pitch	$m = \frac{25.4}{P_d}$
Number of Teeth	Module and Pitch Diameter	$N = \frac{D}{m}$
Addendum	Module	$a = m$
Dedendum	Module	$b = 1.25m$
Outside Diameter	Module and Pitch Diameter or Number of Teeth	$D_o = D + 2m = m(N + 2)$
Root Diameter	Pitch Diameter and Module	$D_R = D - 2.5m$
Base Circle Diameter	Pitch Diameter and Pressure Angle	$D_b = D\cos\phi$
Base Pitch	Module and Pressure Angle	$p_b = m\pi\cos\phi$
Tooth Thickness at Standard Pitch Diameter	Module	$T_{std} = \frac{\pi}{2}m$
Center Distance	Module and Number of Teeth	$C = \frac{m(N_1 + N_2)}{2}$
Contact Ratio	Outside Radii, Base Circle Radii, Center Distance, Pressure Angle	$m_p = \frac{\sqrt{{}_1R_o - {}_1R_b} + \sqrt{{}_2R_o - {}_2R_b} - C\sin\phi}{m\pi\cos\phi}$
Backlash (linear)	Change in Center Distance	$B = 2(\Delta C)\tan\phi$
Backlash (linear)	Change in Tooth Thickness	$B = \Delta T$
Backlash (linear) Along Line-of-action	Linear Backlash Along Pitch Circle	$B_{LA} = B\cos\phi$
Backlash, Angular	Linear Backlash	${}_aB = 6880\frac{B}{D}$ (arc minutes)
Min. No. of Teeth for No Undercutting	Pressure Angle	$N_c = \frac{2}{\sin^2\phi}$

* All linear dimensions in millimeters
Symbols per **Table 1-4**

SECTION 2 INTRODUCTION TO GEAR TECHNOLOGY

This section presents a technical coverage of gear fundamentals. It is intended as a broad coverage written in a manner that is easy to follow and to understand by anyone interested in knowing how gear systems function. Since gearing involves specialty components, it is expected that not all designers and engineers possess or have been exposed to every aspect of this subject. However, for proper use of gear components and design of gear systems it is essential to have a minimum understanding of gear basics and a reference source for details.

For those to whom this is their first encounter with gear components, it is suggested this technical treatise be read in the order presented so as to obtain a logical development of the subject. Subsequently, and for those already familiar with gears, this material can be used selectively in random access as a design reference.

2.1 Basic Geometry Of Spur Gears

The fundamentals of gearing are illustrated through the spur gear tooth, both because it is the simplest, and hence most comprehensible, and because it is the form most widely used, particularly for instruments and control systems.

The basic geometry and nomenclature of a spur gear mesh is shown in **Figure 2-1**. The essential features of a gear mesh are:

1. Center distance.
2. The pitch circle diameters (or pitch diameters).
3. Size of teeth (or module).
4. Number of teeth.
5. Pressure angle of the contacting involutes.

Details of these items along with their interdependence and definitions are covered in subsequent paragraphs.

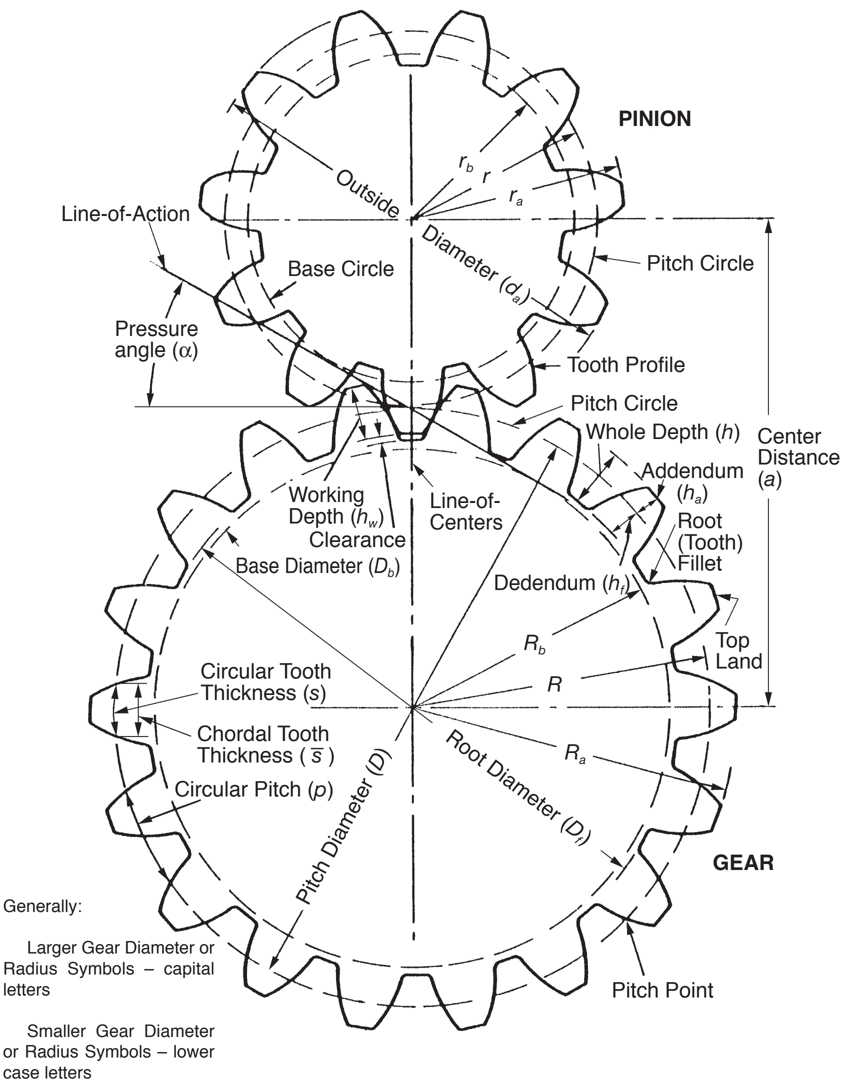


Fig. 2-1 Basic Gear Geometry

2.2 The Law Of Gearing

A primary requirement of gears is the constancy of angular velocities or proportionality of position transmission. Precision instruments require positioning fidelity. High-speed and/or high-power gear trains also require transmission at constant angular velocities in order to avoid severe dynamic problems.

Constant velocity (i.e., constant ratio) motion transmission is defined as "conjugate action" of the gear tooth profiles. A geometric relationship can be derived (2, 12)* for the form of the tooth profiles to provide conjugate action, which is summarized as the Law of Gearing as follows:

"A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point."

Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves.

2.3 The Involute Curve

There is almost an infinite number of curves that can be developed to satisfy the law of gearing, and many different curve forms have been tried in the past. Modern gearing (except for clock gears) is based on involute teeth. This is due to three major advantages of the involute curve:

1. Conjugate action is independent of changes in center distance.
2. The form of the basic rack tooth is straight-sided, and therefore is relatively simple and can be accurately made; as a generating tool it imparts high accuracy to the cut gear tooth.
3. One cutter can generate all gear teeth numbers of the same pitch.

The involute curve is most easily understood as the trace of a point at the end of a taut string that unwinds from a cylinder. It is imagined that a point on a string, which is pulled taut in a fixed direction, projects its trace onto a plane that rotates with the base circle. See **Figure 2-2**. The base cylinder, or base circle as referred to in gear literature, fully defines the form of the involute and in a gear it is an inherent parameter, though invisible.

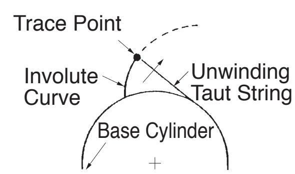


Fig. 2-2 Generation of an Involute by a Taut String

The development and action of mating teeth can be visualized by imagining the taut string as being unwound from one base circle and wound on to the other, as shown in **Figure 2-3a**. Thus, a single point on the string simultaneously traces an involute on each base circle's rotating plane. This pair of involutes is conjugate, since at all points of contact the common normal is the common tangent which passes through a fixed point on the line-of-centers. If a second winding/unwinding taut string is wound around the base circles in the opposite direction, **Figure 2-3b**, oppositely curved involutes are generated which can accommodate motion reversal. When the involute pairs are properly spaced, the result is the involute gear tooth, **Figure 2-3c**.

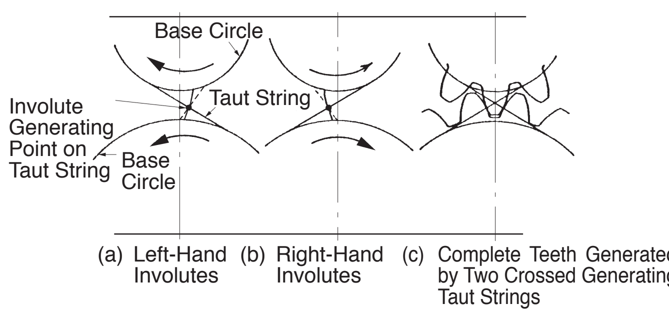


Fig. 2-3 Generation and Action of Gear Teeth

* Numbers in parentheses refer to references at end of text.

2.4 Pitch Circles

Referring to **Figure 2-4**, the tangent to the two base circles is the line of contact, or line-of-action in gear vernacular. Where this line crosses the line-of-centers establishes the pitch point, P. This in turn sets the size of the pitch circles, or as commonly called, the pitch diameters. The ratio of the pitch diameters gives the velocity ratio:

Velocity ratio of gear 2 to gear 1 is:

$$i = \frac{d_1}{d_2} \qquad \textbf{(2-1)}$$

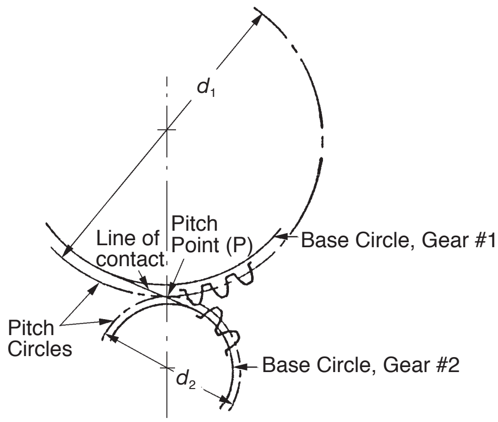


Fig. 2-4 Definition of Pitch Circle and Pitch Point

2.5 Pitch And Module

Essential to prescribing gear geometry is the size, or spacing of the teeth along the pitch circle. This is termed pitch, and there are two basic forms.

Circular pitch — A naturally conceived linear measure along the pitch circle of the tooth spacing. Referring to **Figure 2-5**, it is the linear distance (measured along the pitch circle arc) between corresponding points of adjacent teeth. It is equal to the pitch-circle circumference divided by the number of teeth:

$$p = \text{circular pitch} = \frac{\text{pitch circle circumference}}{\text{number of teeth}} = \frac{\pi d}{z} \qquad \textbf{(2-2)}$$

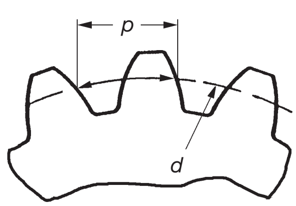


Fig. 2-5 Definition of Circular Pitch

Module — Metric gearing uses the quantity module m in place of the American inch unit, diametral pitch. The module is the length of pitch diameter per tooth. Thus:

$$m = \frac{d}{z} \qquad \textbf{(2-3)}$$

Relation of pitches: From the geometry that defines the two pitches, it can be shown that module and circular pitch are related by the expression:

$$\frac{p}{m} = \pi \qquad \textbf{(2-4)}$$

This relationship is simple to remember and permits an easy transformation from one to the other.

Diametral pitch P_d is widely used in England and America to represent the tooth size. The relation between diametral pitch and module is as follows:

$$m = \frac{25.4}{P_d} \qquad \textbf{(2-5)}$$

2.6 Module Sizes And Standards

Module m represents the size of involute gear tooth. The unit of module is mm. Module is converted to circular pitch p, by the factor π.

$$p = \pi m \qquad \textbf{(2-6)}$$

Table 2-1 is extracted from JIS B 1701-[1973] which defines the tooth profile and dimensions of involute gears. It divides the standard module into three series. **Figure 2-6** shows the comparative size of various rack teeth.

Table 2-1 Standard Values of Module unit: mm

Series 1	Series 2	Series 3	Series 1	Series 2	Series 3
0.1				3.5	
	0.15				3.75
0.2			4		
	0.25			4.5	
0.3			5		
	0.35			5.5	
0.4			6		
	0.45				6.5
0.5				7	
	0.55		8		
0.6				9	
		0.65	10		
	0.7			11	
	0.75		12		
0.8				14	
	0.9		16		
1				18	
1.25			20		
1.5				22	
	1.75		25		
2				28	
	2.25		32		
2.5				36	
	2.75		40		
3				45	
		3.25	50		

Note: The preferred choices are in the series order beginning with 1.

Circular pitch, p, is also used to represent tooth size when a special desired spacing is wanted, such as to get an integral feed in a mechanism. In this case, a circular pitch is chosen that is an integer or a special fractional value. This is often the choice in designing position control systems. Another particular usage is the drive of printing plates to provide a given feed.

Most involute gear teeth have the standard whole depth and a standard pressure angle $\alpha = 20°$. **Figure 2-7** shows the tooth profile of a whole depth standard rack tooth and mating gear. It has an addendum of $h_a = 1m$ and dedendum $h_f \geq 1.25m$. If tooth depth is shorter than whole depth it is called a "stub" tooth; and if deeper than whole depth it is a "high" depth tooth.

The most widely used stub tooth has an addendum $h_a = 0.8m$ and dedendum $h_f = 1m$. Stub teeth have more strength than a whole depth gear, but contact ratio is reduced. On the other hand, a high depth tooth can increase contact ratio, but weakens the tooth.

In the standard involute gear, pitch p times the number of teeth becomes the length of pitch circle:

$$\left.\begin{array}{l} d\pi = \pi m z \\ \text{Pitch diameter d is then:} \\ d = mz \end{array}\right\} \qquad \textbf{(2-7)}$$

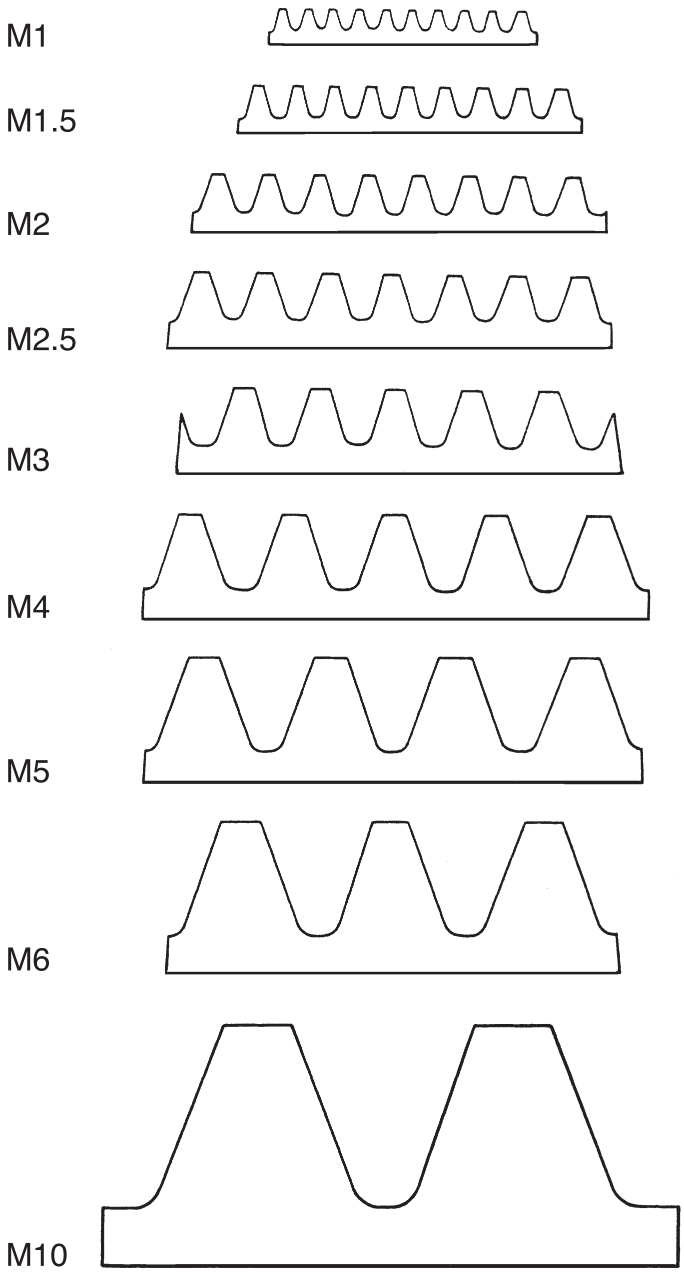


Fig. 2-6 Comparative Size of Various Rack Teeth

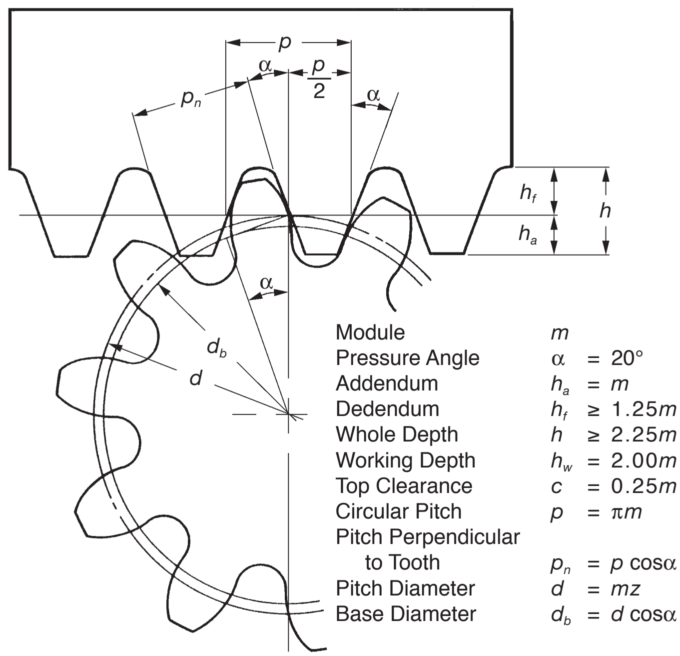


Fig. 2-7 The Tooth Profile and Dimension of Standard Rack

Metric Module and Inch Gear Preferences: Because there is no direct equivalence between the pitches in metric and inch systems, it is not possible to make direct substitutions. Further, there are preferred modules in the metric system. As an aid in using metric gears, **Table 2-2** presents nearest equivalents for both systems, with the preferred sizes in bold type.

Table 2-2 Metric/American Gear Equivalents

Diametral Pitch, *P*	Module, *m*	Circular Pitch		Circular Tooth Thickness		Addendum	
		in	mm	in	mm	in	mm
203.2000	0.125	0.0155	0.393	0.0077	0.196	0.0049	0.125
200	0.12700	0.0157	0.399	0.0079	0.199	0.0050	0.127
180	0.14111	0.0175	0.443	0.0087	0.222	0.0056	0.141
169.333	0.15	0.0186	0.471	0.0093	0.236	0.0059	0.150
150	0.16933	0.0209	0.532	0.0105	0.266	0.0067	0.169
127.000	**0.2**	0.0247	0.628	0.0124	0.314	0.0079	0.200
125	0.20320	0.0251	0.638	0.0126	0.319	0.0080	0.203
120	0.21167	0.0262	0.665	0.0131	0.332	0.0083	0.212
101.600	**0.25**	0.0309	0.785	0.0155	0.393	0.0098	0.250
96	0.26458	0.0327	0.831	0.0164	0.416	0.0104	0.265
92.3636	0.275	0.0340	0.864	0.0170	0.432	0.0108	0.275
84.6667	**0.3**	0.0371	0.942	0.0186	0.471	0.0118	0.300
80	0.31750	0.0393	0.997	0.0196	0.499	0.0125	0.318
78.1538	0.325	0.0402	1.021	0.0201	0.511	0.0128	0.325
72.5714	0.35	0.0433	1.100	0.0216	0.550	0.0138	0.350
72	0.35278	0.0436	1.108	0.0218	0.554	0.0139	0.353
67.733	0.375	0.0464	1.178	0.0232	0.589	0.0148	0.375
64	0.39688	0.0491	1.247	0.0245	0.623	0.0156	0.397
63.500	**0.4**	0.0495	1.257	0.0247	0.628	0.0157	0.400
50.800	**0.5**	0.0618	1.571	0.0309	0.785	0.0197	0.500
50	0.50800	0.0628	1.596	0.0314	0.798	0.0200	0.508
48	0.52917	0.0655	1.662	0.0327	0.831	0.0208	0.529
44	0.57727	0.0714	1.814	0.0357	0.907	0.0227	0.577
42.333	**0.6**	0.0742	1.885	0.0371	0.942	0.0236	0.600
40	0.63500	0.0785	1.995	0.0393	0.997	0.0250	0.635
36.2857	**0.7**	0.0866	2.199	0.0433	1.100	0.0276	0.700
36	0.70556	0.0873	2.217	0.0436	1.108	0.0278	0.706
33.8667	0.75	0.0928	2.356	0.0464	1.178	0.0295	0.750
32	0.79375	0.0982	2.494	0.0491	1.247	0.0313	0.794
31.7500	**0.8**	0.0989	2.513	0.0495	1.257	0.0315	0.800
30	0.84667	0.1047	2.660	0.0524	1.330	0.0333	0.847
28.2222	0.9	0.1113	2.827	0.0557	1.414	0.0354	0.900
28	0.90714	0.1122	2.850	0.0561	1.425	0.0357	0.907
25.4000	**1**	0.1237	3.142	0.0618	1.571	0.0394	1.000
24	1.0583	0.1309	3.325	0.0654	1.662	0.0417	1.058
22	1.1545	0.1428	3.627	0.0714	1.813	0.0455	1.155
20.3200	**1.25**	0.1546	3.927	0.0773	1.963	0.0492	1.250
20	1.2700	0.1571	3.990	0.0785	1.995	0.0500	1.270
18	1.4111	0.1745	4.433	0.0873	2.217	0.0556	1.411
16.9333	**1.5**	0.1855	4.712	0.0928	2.356	0.0591	1.500
16	1.5875	0.1963	4.987	0.0982	2.494	0.0625	1.588
15	1.6933	0.2094	5.320	0.1047	2.660	0.0667	1.693
14.5143	1.75	0.2164	5.498	0.1082	2.749	0.0689	1.750
14	1.8143	0.2244	5.700	0.1122	2.850	0.0714	1.814
13	1.9538	0.2417	6.138	0.1208	3.069	0.0769	1.954
12.7000	**2**	0.2474	6.283	0.1237	3.142	0.0787	2.000
12	2.1167	0.2618	6.650	0.1309	3.325	0.0833	2.117
11.2889	2.25	0.2783	7.069	0.1391	3.534	0.0886	2.250
11	2.3091	0.2856	7.254	0.1428	3.627	0.0909	2.309
10.1600	**2.50**	0.3092	7.854	0.1546	3.927	0.0984	2.500
10	2.5400	0.3142	7.980	0.1571	3.990	0.1000	2.540
9.2364	2.75	0.3401	8.639	0.1701	4.320	0.1083	2.750
9	2.8222	0.3491	8.866	0.1745	4.433	0.1111	2.822
8.4667	**3**	0.3711	9.425	0.1855	4.712	0.1181	3.000
8	3.1750	0.3927	9.975	0.1963	4.987	0.1250	3.175
7.8154	3.25	0.4020	10.210	0.2010	5.105	0.1280	3.250
7.2571	3.5	0.4329	10.996	0.2164	5.498	0.1378	3.500
7	3.6286	0.4488	11.400	0.2244	5.700	0.1429	3.629
6.7733	3.75	0.4638	11.781	0.2319	5.890	0.1476	3.750
6.3500	**4**	0.4947	12.566	0.2474	6.283	0.1575	4.000
6	4.2333	0.5236	13.299	0.2618	6.650	0.1667	4.233
5.6444	4.5	0.5566	14.137	0.2783	7.069	0.1772	4.500
5.3474	4.75	0.5875	14.923	0.2938	7.461	0.1870	4.750
5.0800	**5**	0.6184	15.708	0.3092	7.854	0.1969	5.000
5	5.0800	0.6283	15.959	0.3142	7.980	0.2000	5.080
4.6182	5.5000	0.6803	17.279	0.3401	8.639	0.2165	5.500
4.2333	**6**	0.7421	18.850	0.3711	9.425	0.2362	6.000
4	6.3500	0.7854	19.949	0.3927	9.975	0.2500	6.350
3.9077	6.5000	0.8040	20.420	0.4020	10.210	0.2559	6.500
3.6286	7	0.8658	21.991	0.4329	10.996	0.2756	7.000
3.5000	7.2571	0.8976	22.799	0.4488	11.399	0.2857	7.257
3.1750	**8**	0.9895	25.133	0.4947	12.566	0.3150	8.000
3.1416	8.0851	1.0000	25.400	0.5000	12.700	0.3183	8.085
3	8.4667	1.0472	26.599	0.5236	13.299	0.3333	8.467
2.8222	9	1.1132	28.274	0.5566	14.137	0.3543	9.000
2.5400	**10**	1.2368	31.416	0.6184	15.708	0.3937	10.000
2.5000	10.160	1.2566	31.919	0.6283	15.959	0.4000	10.160
2.3091	11	1.3605	34.558	0.6803	17.279	0.4331	11.000
2.1167	**12**	1.4842	37.699	0.7421	18.850	0.4724	12.000
2	12.700	1.5708	39.898	0.7854	19.949	0.5000	12.700
1.8143	14	1.7316	43.982	0.8658	21.991	0.5512	14.000
1.5875	**16**	1.9790	50.265	0.9895	25.133	0.6299	16.000
1.5000	16.933	2.0944	53.198	1.0472	26.599	0.6667	16.933
1.4111	18	2.2263	56.549	1.1132	28.274	0.7087	18.000
1.2700	**20**	2.4737	62.832	1.2368	31.416	0.7874	20.000
1.1545	22	2.7211	69.115	1.3605	34.558	0.8661	22.000
1.0583	24	2.9684	75.398	1.4842	37.699	0.9449	24.000
1.0160	**25**	3.0921	78.540	1.5461	39.270	0.9843	25.000
1	25.400	3.1416	79.796	1.5708	39.898	1.0000	25.400
0.9407	27	3.3395	84.823	1.6697	42.412	1.0630	27.000
0.9071	28	3.4632	87.965	1.7316	43.982	1.1024	28.000
0.8467	30	3.7105	94.248	1.8553	47.124	1.1811	30.000
0.7938	**32**	3.9579	100.531	1.9790	50.265	1.2598	32.000
0.7697	33	4.0816	103.673	2.0408	51.836	1.2992	33.000
0.7500	33.867	4.1888	106.395	2.0944	53.198	1.3333	33.867
0.7056	36	4.4527	113.097	2.2263	56.549	1.4173	36.000
0.6513	39	4.8237	122.522	2.4119	61.261	1.5354	39.000
0.6350	**40**	4.9474	125.664	2.4737	62.832	1.5748	40.000
0.6048	42	5.1948	131.947	2.5974	65.973	1.6535	42.000
0.5644	45	5.5658	141.372	2.7829	70.686	1.7717	45.000
0.5080	**50**	6.1842	157.080	3.0921	78.540	1.9685	50.000
0.5000	50.800	6.2832	159.593	3.1416	79.796	2.0000	50.800

NOTE: Bold face diametral pitches and modules designate preferred values.

2.7 Gear Types And Axial Arrangements

In accordance with the orientation of axes, there are three categories of gears:

1. **Parallel Axes Gears**
2. **Intersecting Axes Gears**
3. **Nonparallel and Nonintersecting Axes Gears**

Spur and helical gears are the parallel axes gears. Bevel gears are the intersecting axes gears. Screw or crossed helical, worm and hypoid gears handle the third category. **Table 2-3** lists the gear types per axes orientation.

Also, included in **Table 2-3** is the theoretical efficiency range of the various gear types. These figures do not include bearing and lubricant losses. Also, they assume ideal mounting in regard to axis orientation and center distance. Inclusion of these realistic considerations will downgrade the efficiency numbers.

Table 2-3 Types of Gears and Their Categories

Categories of Gears	Types of Gears	Efficiency (%)
Parallel Axes Gears	Spur Gear Spur Rack Internal Gear Helical Gear Helical Rack Double Helical Gear	98 ... 99.5
Intersecting Axes Gears	Straight Bevel Gear Spiral Bevel Gear Zerol Gear	98 ... 99
Nonparallel and Nonintersecting Axes Gears	Worm Gear Screw Gear Hypoid Gear	30 ... 90 70 ... 95 96 ... 98

2.7.1 Parallel Axes Gears

1. Spur Gear

This is a cylindrical shaped gear in which the teeth are parallel to the axis. It has the largest applications and, also, it is the easiest to manufacture.

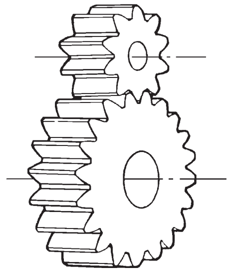


Fig. 2-8 Spur Gear

2. Spur Rack

This is a linear shaped gear which can mesh with a spur gear with any number of teeth. The spur rack is a portion of a spur gear with an infinite radius.

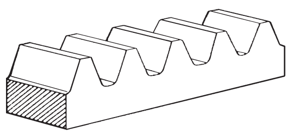


Fig. 2-9 Spur Rack

3. Internal Gear

This is a cylindrical shaped gear but with the teeth inside the circular ring. It can mesh with a spur gear. Internal gears are often used in planetary gear systems.

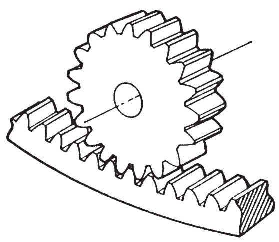


Fig. 2-10 Internal Gear and Spur Gear

4. Helical Gear

This is a cylindrical shaped gear with helicoid teeth. Helical gears can bear more load than spur gears, and work more quietly. They are widely used in industry. A disadvantage is the axial thrust force the helix form causes.

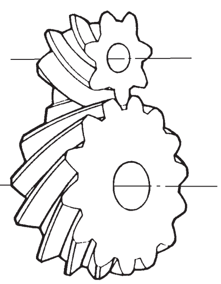


Fig. 2-11 Helical Gear

5. Helical Rack

This is a linear shaped gear which meshes with a helical gear. Again, it can be regarded as a portion of a helical gear with infinite radius.

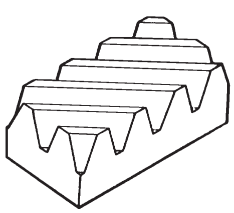


Fig. 2-12 Helical Rack

6. Double Helical Gear

This is a gear with both left-hand and right-hand helical teeth. The double helical form balances the inherent thrust forces.

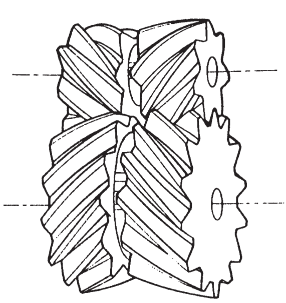


Fig. 2-13 Double Helical Gear

2.7.2 Intersecting Axes Gears

1. Straight Bevel Gear

This is a gear in which the teeth have tapered conical elements that have the same direction as the pitch cone base line (generatrix). The straight bevel gear is both the simplest to produce and the most widely applied in the bevel gear family.

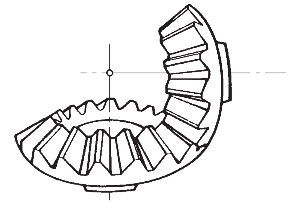


Fig. 2-14 Straight Bevel Gear

2. Spiral Bevel Gear

This is a bevel gear with a helical angle of spiral teeth. It is much more complex to manufacture, but offers a higher strength and lower noise.

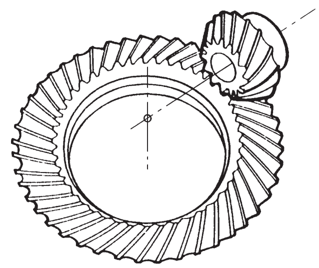


Fig. 2-15 Spiral Bevel Gear

3. Zerol Gear

Zerol gear is a special case of spiral bevel gear. It is a spiral bevel with zero degree of spiral angle tooth advance. It has the characteristics of both the straight and spiral bevel gears. The forces acting upon the tooth are the same as for a straight bevel gear.

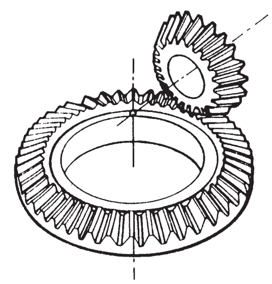


Fig. 2-16 Zerol Gear

2.7.3 Nonparallel And Nonintersecting Axes Gears

1. Worm And Worm Gear

Worm set is the name for a meshed worm and worm gear. The worm resembles a screw thread; and the mating worm gear a helical gear, except that it is made to envelope the worm as seen along the worm's axis. The outstanding feature is that the worm offers a very large gear ratio in a single mesh. However, transmission efficiency is very poor due to a great amount of sliding as the worm tooth engages with its mating worm gear tooth and forces rotation by pushing and sliding. With proper choices of materials and lubrication, wear can be contained and noise is reduced.

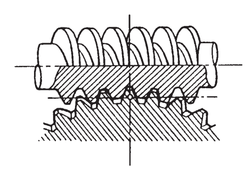


Fig. 2-17 Worm Gear

2. Screw Gear (Crossed Helical Gear)

Two helical gears of opposite helix angle will mesh if their axes are crossed. As separate gear components, they are merely conventional helical gears. Installation on crossed axes converts them to screw gears. They offer a simple means of gearing skew axes at any angle. Because they have point contact, their load carrying capacity is very limited.

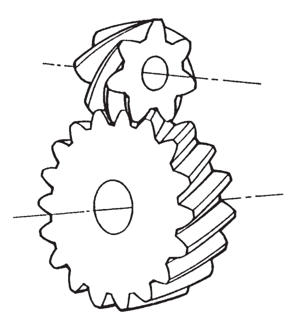


Fig. 2-18 Screw Gear

2.7.4 Other Special Gears

1. Face Gear

This is a pseudobevel gear that is limited to 90° intersecting axes. The face gear is a circular disc with a ring of teeth cut in its side face; hence the name face gear. Tooth elements are tapered towards its center. The mate is an ordinary spur gear. It offers no advantages over the standard bevel gear, except that it can be fabricated on an ordinary shaper gear generating machine.

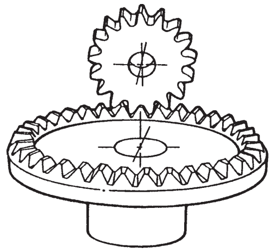


Fig. 2-19 Face Gear

2. Double Enveloping Worm Gear

This worm set uses a special worm shape in that it partially envelops the worm gear as viewed in the direction of the worm gear axis. Its big advantage over the standard worm is much higher load capacity. However, the worm gear is very complicated to design and produce, and sources for manufacture are few.

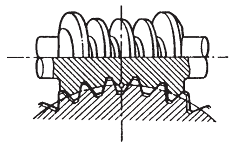


Fig. 2-20 Double Enveloping Worm Gear

3. Hypoid Gear

This is a deviation from a bevel gear that originated as a special development for the automobile industry. This permitted the drive to the rear axle to be nonintersecting, and thus allowed the auto body to be lowered. It looks very much like the spiral bevel gear. However, it is complicated to design and is the most difficult to produce on a bevel gear generator.

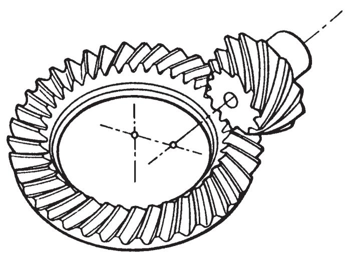


Fig. 2-21 Hypoid Gear

SECTION 3 DETAILS OF INVOLUTE GEARING

3.1 Pressure Angle

The pressure angle is defined as the angle between the line-of-action (common tangent to the base circles in **Figures 2-3** and **2-4**) and a perpendicular to the line-of-centers. See **Figure 3-1**. From the geometry of these figures, it is obvious that the pressure angle varies (slightly) as the center distance of a gear pair is altered. The base circle is related to the pressure angle and pitch diameter by the equation:

$$d_b = d \cos\alpha \qquad \textbf{(3-1)}$$

where d and α are the standard values, or alternately:

$$d_b = d' \cos\alpha' \qquad \textbf{(3-2)}$$

where d' and α' are the exact operating values.

The basic formula shows that the larger the pressure angle the smaller the base circle. Thus, for standard gears, 14.5° pressure angle gears have base circles much nearer to the roots of teeth than 20° gears. It is for this reason that 14.5° gears encounter greater undercutting problems than 20° gears. This is further elaborated on in **SECTION 4.3**.

3.2 Proper Meshing And Contact Ratio

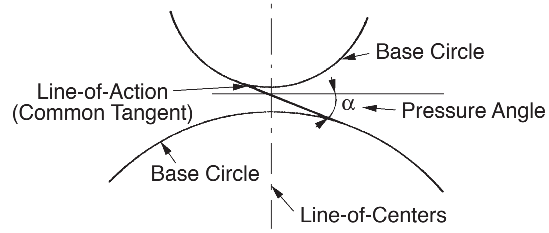


Fig. 3-1 Definition of Pressure Angle

Figure 3-2 shows a pair of standard gears meshing together. The contact point of the two involutes, as **Figure 3-2** shows, slides along the common tangent of the two base circles as rotation occurs. The common tangent is called the line-of-contact, or line-of-action.

A pair of gears can only mesh correctly if the pitches and the pressure angles are the same. Pitch comparison can be module m, circular p, or base p_b.

That the pressure angles must be identical becomes obvious from the following equation for base pitch:

$$p_b = \pi m \cos\alpha \qquad \textbf{(3-3)}$$

Thus, if the pressure angles are different, the base pitches cannot be identical.

The length of the line-of-action is shown as ab in **Figure 3-2**.

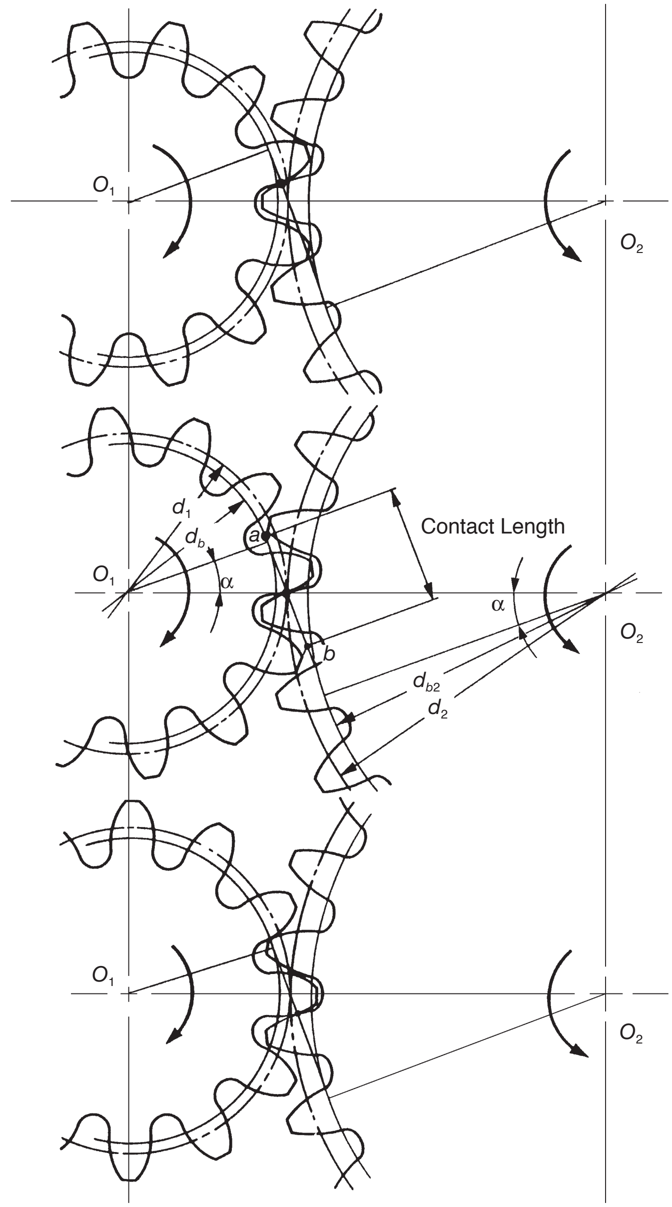


Fig. 3-2 The Meshing of Involute Gear

3.2.1 Contact Ratio

To assure smooth continuous tooth action, as one pair of teeth ceases contact a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as possible. The measure of this overlapping is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. **Figure 3-3** shows the geometry. The length-of-action is determined from the intersection of the line-of-action and the outside radii. For the simple case of a pair of spur gears, the ratio of the length-of-action to the base pitch is determined from:

$$\varepsilon_\gamma = \frac{\sqrt{(R_a^2 - R_b^2)} + \sqrt{(r_a^2 - r_b^2)} - a\sin\alpha}{p\cos\alpha} \qquad (3\text{-}4)$$

It is good practice to maintain a contact ratio of 1.2 or greater. Under no circumstances should the ratio drop below 1.1, calculated for all tolerances at their worst-case values.

A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact. Such a high contact ratio generally is not obtained with external spur gears, but can be developed in the meshing of an internal and external spur gear pair or specially designed nonstandard external spur gears.

More detail is presented about contact ratio, including calculation equations for specific gear types, in **SECTION 11**.

3.3 The Involute Function

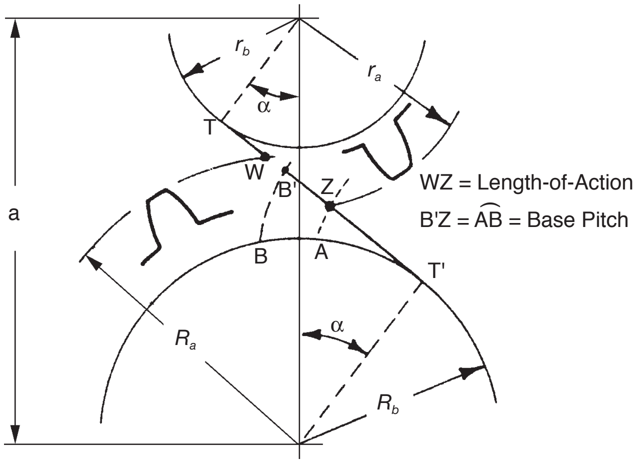


Fig. 3-3 Geometry of Contact Ratio

Figure 3-4 shows an element of involute curve. The definition of involute curve is the curve traced by a point on a straight line which rolls without slipping on the circle. The circle is called the base circle of the involutes. Two opposite hand involute curves meeting at a cusp form a gear tooth curve. We can see, from **Figure 3-4**, the length of base circle arc ac equals the length of straight line bc.

$$\tan\alpha = \frac{bc}{Oc} = \frac{r_b\theta}{r_b} = \theta \text{ (radian)} \qquad (3\text{-}5)$$

The θ in **Figure 3-4** can be expressed as invα + α, then **Formula (3-5)** will become:

$$\text{inv}\alpha = \tan\alpha - \alpha \qquad (3\text{-}6)$$

Function of α, or invα, is known as involute function. Involute function is very important in gear design. Involute function values can be obtained from appropriate tables. With the center of the base circle *O* at the origin of a coordinate system, the involute curve can be expressed by values of *x* and *y* as follows:

$$\left.\begin{aligned} x &= r\cos(\text{inv}\alpha) = \frac{r_b}{\cos\alpha}\cos(\text{inv}\alpha) \\ y &= r\sin(\text{inv}\alpha) = \frac{r_b}{\cos\alpha}\sin(\text{inv}\alpha) \end{aligned}\right\} \qquad (3\text{-}7)$$

where, $r = \dfrac{r_b}{\cos\alpha}$.

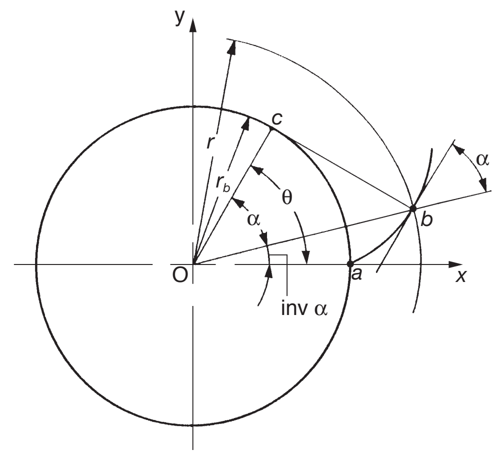


Fig. 3-4 The Involute Curve

SECTION 4 SPUR GEAR CALCULATIONS

4.1 Standard Spur Gear

Figure 4-1 shows the meshing of standard spur gears. The meshing of standard spur gears means pitch circles of two gears contact and roll with each other. The calculation formulas are in **Table 4-1**.

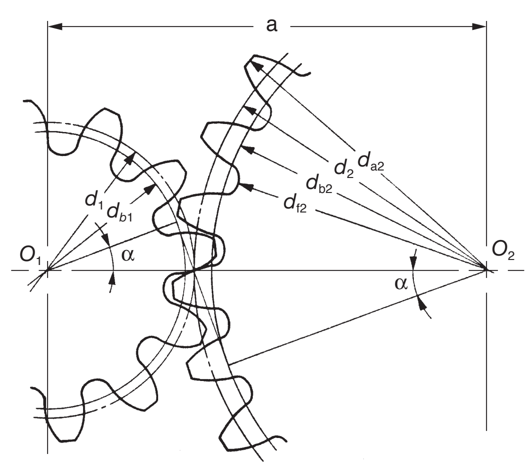


Fig. 4-1 The Meshing of Standard Spur Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = x_2 = 0$)

Table 4-1 The Calculation of Standard Spur Gears

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2 *		12	24
4	Center Distance	a	$\frac{(z_1 + z_2)m}{2}$*	54.000	
5	Pitch Diameter	d	zm	36.000	72.000
6	Base Diameter	d_b	$d \cos\alpha$	33.829	67.658
7	Addendum	h_a	$1.00m$	3.000	
8	Dedendum	h_f	$1.25m$	3.750	
9	Outside Diameter	d_a	$d + 2m$	42.000	78.000
10	Root Diameter	d_f	$d - 2.5m$	28.500	64.500

* The subscripts 1 and 2 of z_1 and z_2 denote pinion and gear.

All calculated values in **Table 4-1** are based upon given module m and number of teeth z_1 and z_2. If instead module m, center distance a and speed ratio i are given, then the number of teeth, z_1 and z_2, would be calculated with the formulas as shown in **Table 4-2**.

Table 4-2 The Calculation of Teeth Number

No.	Item	Symbol	Formula		Example	
1	Module	m			3	
2	Center Distance	a			54.000	
3	Speed Ratio	i			0.8	
4	Sum of No. of Teeth	$z_1 + z_2$	$\frac{2a}{m}$		36	
5	Number of Teeth	$z_1\ ,\ z_2$	$\frac{i(z_1 + z_2)}{i + 1}$	$\frac{(z_1 + z_2)}{i + 1}$	16	20

Note that the numbers of teeth probably will not be integer values by calculation with the formulas in **Table 4-2**. Then it is incumbent upon the designer to choose a set of integer numbers of teeth that are as close as possible to the theoretical values. This will likely result in both slightly changed gear ratio and center distance. Should the center distance be inviolable, it will then be necessary to resort to profile shifting. This will be discussed later in this section.

4.2 The Generating Of A Spur Gear

Involute gears can be readily generated by rack type cutters. The hob is in effect a rack cutter. Gear generation is also accomplished with gear type cutters using a shaper or planer machine.

Figure 4-2 illustrates how an involute gear tooth profile is generated. It shows how the pitch line of a rack cutter rolling on a pitch circle generates a spur gear.

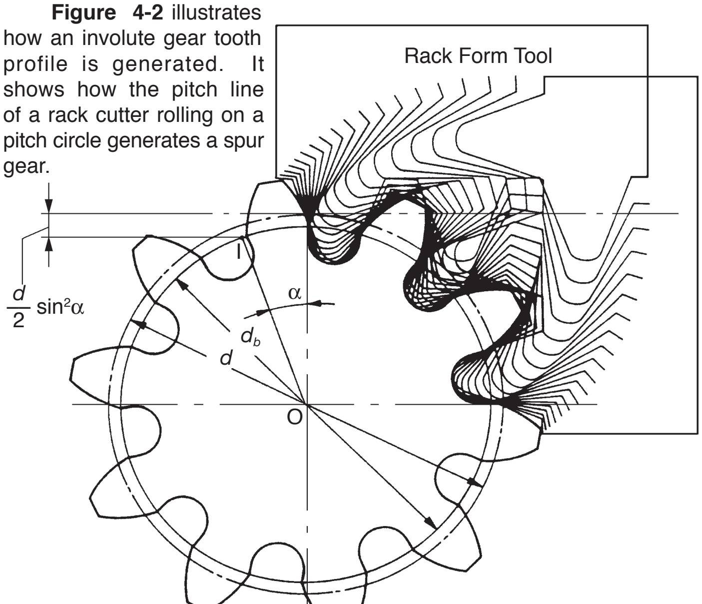


Fig. 4-2 The Generating of a Standard Spur Gear ($\alpha = 20°, z = 10, x = 0$)

4.3 Undercutting

From **Figure 4-3**, it can be seen that the maximum length of the line-of-contact is limited to the length of the common tangent. Any tooth addendum that extends beyond the tangent points (T and T') is not only useless, but interferes with the root fillet area of the mating tooth. This results in the typical undercut tooth, shown in **Figure 4-4**. The undercut not only weakens the tooth with a wasp-like waist, but also removes some of the useful involute adjacent to the base circle.

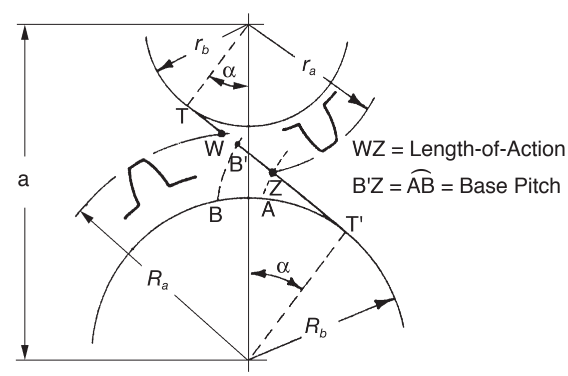


Fig. 4-3 Geometry of Contact Ratio

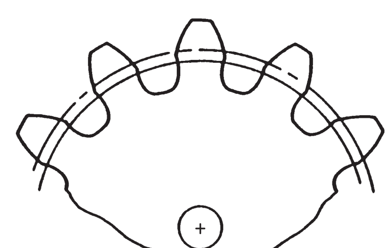


Fig. 4-4 Example of Undercut Standard Design Gear
(12 Teeth, 20° Pressure Angle)

From the geometry of the limiting length-of-contact (T-T', **Figure 4-3**), it is evident that interference is first encountered by the addenda of the gear teeth digging into the mating-pinion tooth flanks. Since addenda are standardized by a fixed value ($h_a = m$), the interference condition becomes more severe as the number of teeth on the mating gear increases. The limit is reached when the gear becomes a rack. This is a realistic case since the hob is a rack-type cutter. The result is that standard gears with teeth numbers below a critical value are automatically undercut in the generating process. The condition for no undercutting in a standard spur gear is given by the expression:

$$\left.\begin{aligned} &\text{Max addendum} = h_a \leq \frac{mz}{2}\sin^2\alpha \\ &\text{and the minimum number of teeth is:} \\ &z_c \geq \frac{2}{\sin^2\alpha} \end{aligned}\right\} \qquad (\mathbf{4\text{-}1})$$

This indicates that the minimum number of teeth free of undercutting decreases with increasing pressure angle. For 14.5° the value of z_c is 32, and for 20° it is 18. Thus, 20° pressure angle gears with low numbers of teeth have the advantage of much less undercutting and, therefore, are both stronger and smoother acting.

4.4 Enlarged Pinions

Undercutting of pinion teeth is undesirable because of losses of strength, contact ratio and smoothness of action. The severity of these faults depends upon how far below z_c the teeth number is. Undercutting for the first few numbers is small and in many applications its adverse effects can be neglected.

For very small numbers of teeth, such as ten and smaller, and for high-precision applications, undercutting should be avoided. This is achieved by pinion enlargement (or correction as often termed), wherein the pinion teeth, still generated with a standard cutter, are shifted radially outward to form a full involute tooth free of undercut. The tooth is enlarged both radially and circumferentially. Comparison of a tooth form before and after enlargement is shown in **Figure 4-5**.

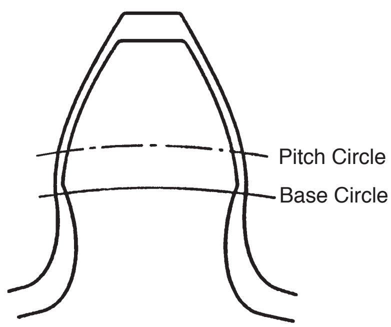


Fig. 4-5 Comparison of Enlarged and Undercut Standard Pinion
(13 Teeth, 20° Pressure Angle, Fine Pitch Standard)

4.5 Profile Shifting

As **Figure 4-2** shows, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut volume. To prevent undercut, a positive correction must be introduced. A positive correction, as in **Figure 4-6**, can prevent undercut.

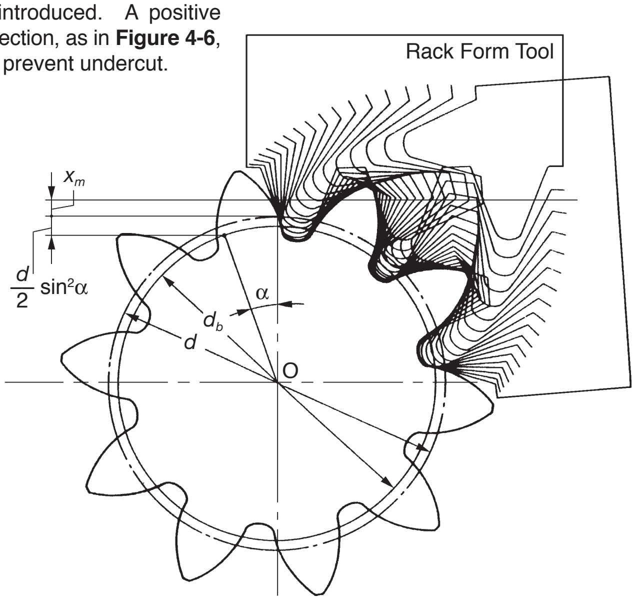


Fig. 4-6 Generating of Positive Shifted Spur Gear
(α = 20°, z = 10, x = +0.5)

Undercutting will get worse if a negative correction is applied. See **Figure 4-7**.

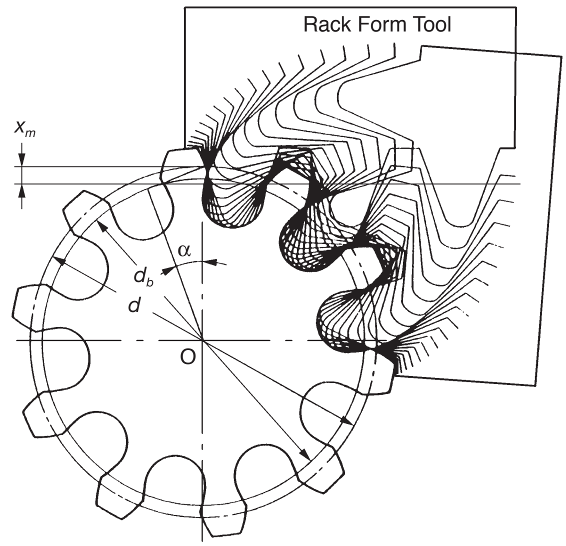


Fig. 4-7 The Generating of Negative Shifted Spur Gear
(α = 20°, z = 10, x = -0.5)

The extra feed of gear cutter (xm) in **Figures 4-6** and **4-7** is the amount of shift or correction. And x is the shift coefficient.

The condition to prevent undercut in a spur gear is:

$$m - xm \leq \frac{zm}{2}\sin^2\alpha \qquad (\mathbf{4\text{-}2})$$

The number of teeth without undercut will be:

$$z_c = \frac{2(1 - x)}{\sin^2\alpha} \qquad (\mathbf{4\text{-}3})$$

The coefficient without undercut is:

$$x = 1 - \frac{z_c}{2}\sin^2\alpha \qquad (\mathbf{4\text{-}4})$$

Profile shift is not merely used to prevent undercut. It can be used to adjust center distance between two gears.

If a positive correction is applied, such as to prevent undercut in a pinion, the tooth thickness at top is thinner.

Table 4-3 presents the calculation of top land thickness.

Table 4-3 The Calculations of Top Land Thickness

No.	Item	Symbol	Formula	Example
1	Pressure angle at outside circle of gear	α_a	$\cos^{-1}\left(\frac{d_b}{d_a}\right)$	m = 2, α = 20°, z = 16, x = +0.3, d = 32, d_b = 30.07016 d_a = 37.2 α_a = 36.06616° invα_a = 0.098835 invα = 0.014904 θ = 1.59815° (0.027893 radian) s_a = 1.03762
2	Half of top land angle of outside circle	θ	$\frac{\pi}{2z} + \frac{2x\tan\alpha}{z} + (\text{inv}\alpha - \text{inv}\alpha_a)$ (radian)	
3	Top land thickness	s_a	θd_a	

4.6 Profile Shifted Spur Gear

Figure 4-8 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters d_w and the working (operating) pressure angle α_w.

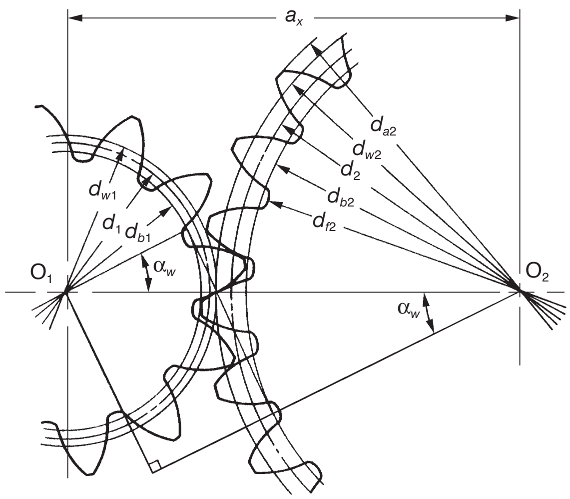


Fig. 4-8 The Meshing of Profile Shifted Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

These values are obtainable from the operating (or i.e., actual) center distance and the following formulas:

$$\left.\begin{aligned} d_{w1} &= 2a_x \frac{z_1}{z_1 + z_2} \\ d_{w2} &= 2a_x \frac{z_2}{z_1 + z_2} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a_x}\right) \end{aligned}\right\} \quad \textbf{(4-5)}$$

In the meshing of profile shifted gears, it is the operating pitch circles that are in contact and roll on each other that portrays gear action. The standard pitch circles no longer are of significance; and the operating pressure angle is what matters.

A standard spur gear is, according to **Table 4-4**, a profile shifted gear with 0 coefficient of shift; that is, $x_1 = x_2 = 0$.

Table 4-5 is the inverse formula of items from 4 to 8 of **Table 4-4**.

There are several theories concerning how to distribute the sum of coefficient of profile shift, $x_1 + x_2$, into pinion, x_1, and gear, x_2, separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

Table 4-4 The Calculation of Positive Shifted Gear (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		12	24
4	Coefficient of Profile Shift	x_1, x_2		0.6	0.36
5	Involute Function α_w	inv α_w	$2\tan\alpha\left(\frac{x_1 + x_2}{z_1 + z_2}\right) + \text{inv}\,\alpha$	0.034316	
6	Working Pressure Angle	α_w	Find from Involute Function Table	26.0886°	
7	Center Distance Increment Factor	y	$\frac{z_1 + z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.83329	
8	Center Distance	a_x	$\left(\frac{z_1 + z_2}{2} + y\right)m$	56.4999	
9	Pitch Diameter	d	zm	36.000	72.000
10	Base Diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1 + y - x_2)m$ $(1 + y - x_1)m$	4.420	3.700
13	Whole Depth	h	$[2.25 + y - (x_1 + x_2)]m$	6.370	
14	Outside Diameter	d_a	$d + 2h_a$	44.840	79.400
15	Root Diameter	d_f	$d_a - 2h$	32.100	66.660

Table 4-5 The Calculation of Positive Shifted Gear (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		56.4999	
2	Center Distance Increment Factor	y	$\frac{a_x}{m} - \frac{z_1 + z_2}{2}$	0.8333	
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{(z_1 + z_2)\cos\alpha}{2y + z_1 + z_2}\right]$	26.0886°	
4	Sum of Coefficient of Profile Shift	$x_1 + x_2$	$\frac{(z_1 + z_2)(\text{inv}\,\alpha_w - \text{inv}\,\alpha)}{2\tan\alpha}$	0.9600	
5	Coefficient of Profile Shift	x_1 , x_2		0.6000	0.3600

4.7 Rack And Spur Gear

Table 4-6 presents the method for calculating the mesh of a rack and spur gear. **Figure 4-9a** shows the pitch circle of a standard gear and the pitch line of the rack.

One rotation of the spur gear will displace the rack l one circumferential length of the gear's pitch circle, per the formula:

$$l = \pi m z \qquad \textbf{(4-6)}$$

Figure 4-9b shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.

Table 4-6 presents the calculation of a meshed profile shifted spur gear and rack. If the correction factor x_1 is 0, then it is the case of a standard gear meshed with the rack.

The rack displacement, l, is not changed in any way by the profile shifting. **Equation (4-6)** remains applicable for any amount of profile shift.

Table 4-6 The Calculation of Dimensions of a Profile Shifted Spur Gear and a Rack

No.	Item	Symbol	Formula	Example	
				Spur Gear	Rack
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z		12	—
4	Coefficient of Profile Shift	x		0.6	
5	Height of Pitch Line	H		—	32.000
6	Working Pressure Angle	α_w		20°	
7	Center Distance	a_x	$\frac{zm}{2} + H + xm$	51.800	
8	Pitch Diameter	d	zm	36.000	—
9	Base Diameter	d_b	$d \cos\alpha$	33.829	
10	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	36.000	
11	Addendum	h_a	$m(1 + x)$	4.800	3.000
12	Whole Depth	h	$2.25m$	6.750	
13	Outside Diameter	d_a	$d + 2h_a$	45.600	—
14	Root Diameter	d_f	$d_a - 2h$	32.100	

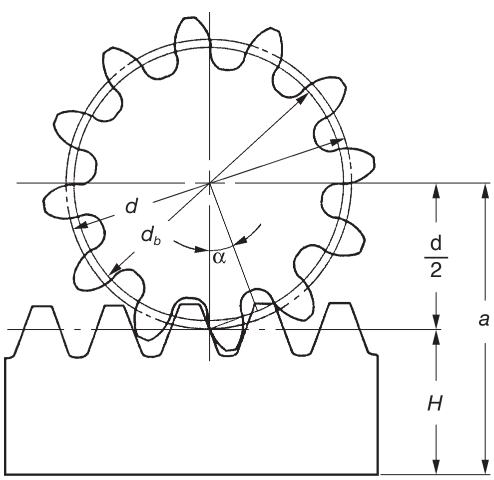


Fig. 4-9a The Meshing of Standard Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = 0$)

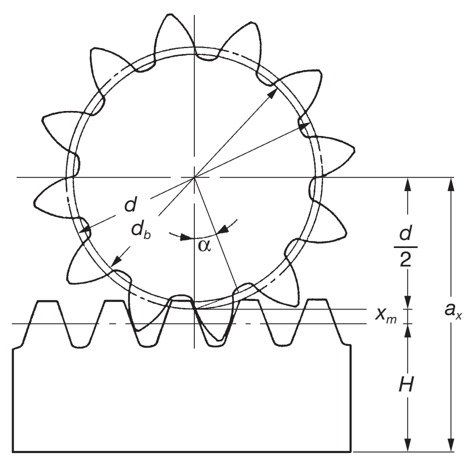


Fig. 4-9b The Meshing of Profile Shifted Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = +0.6$)

SECTION 5 INTERNAL GEARS

5.1 Internal Gear Calculations

Calculation of a Profile Shifted Internal Gear

Figure 5-1 presents the mesh of an internal gear and external gear. Of vital importance is the operating (working) pitch diameters, d_w, and operating (working) pressure angle, α_w. They can be derived from center distance, a_x, and **Equations (5-1)**.

$$\left.\begin{aligned} d_{w1} &= 2a_x\left(\frac{z_1}{z_2 - z_1}\right) \\ d_{w2} &= 2a_x\left(\frac{z_2}{z_2 - z_1}\right) \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b2} - d_{b1}}{2a_x}\right) \end{aligned}\right\} \qquad \textbf{(5-1)}$$

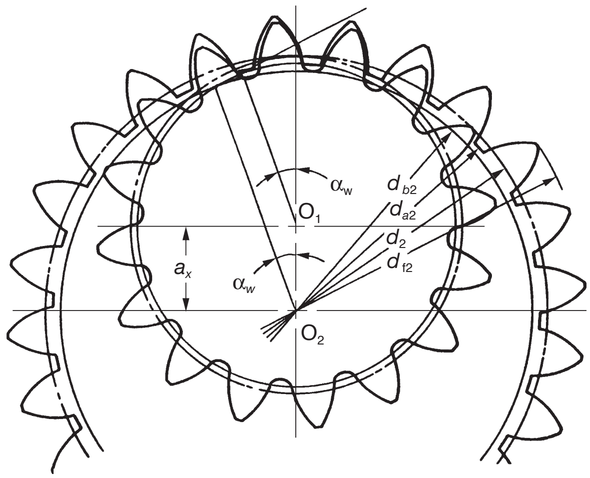


Fig. 5-1 The Meshing of Internal Gear and External Gear
($\alpha = 20°$, $z_1 = 16$, $z_2 = 24$, $x_1 = x_2 = 0.5$)

Table 5-1 shows the calculation steps. It will become a standard gear calculation if $x_1 = x_2 = 0$.

If the center distance, a_x, is given, x_1 and x_2 would be obtained from the inverse calculation from item 4 to item 8 of **Table 5-1**. These inverse formulas are in **Table 5-2**.

Pinion cutters are often used in cutting internal gears and external gears. The actual value of tooth depth and root diameter, after cutting, will be slightly different from the calculation. That is because the cutter has a coefficient of shifted profile. In order to get a correct tooth profile, the coefficient of cutter should be taken into consideration.

Table 5-1 The Calculation of a Profile Shifted Internal Gear and External Gear (1)

No.	Item	Symbol	Formula	Example External Gear (1)	Example Internal Gear (2)
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		16	24
4	Coefficient of Profile Shift	x_1, x_2		0	0.5
5	Involute Function α_w	$\text{inv}\alpha_w$	$2\tan\alpha\left(\dfrac{x_2 - x_1}{z_2 - z_1}\right) + \text{inv}\alpha$	0.060401	
6	Working Pressure Angle	α_w	Find from Involute Function Table	31.0937°	
7	Center Distance Increment Factor	y	$\dfrac{z_2 - z_1}{2}\left(\dfrac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.389426	
8	Center Distance	a_x	$\left(\dfrac{z_2 - z_1}{2} + y\right)m$	13.1683	
9	Pitch Diameter	d	zm	48.000	72.000
10	Base Circle Diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working Pitch Diameter	d_w	$\dfrac{d_b}{\cos\alpha_w}$	52.673	79.010
12	Addendum	h_{a1} h_{a2}	$(1 + x_1)m$ $(1 - x_2)m$	3.000	1.500
13	Whole Depth	h	$2.25m$	6.75	
14	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 - 2h_{a2}$	54.000	69.000
15	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{a2} + 2h$	40.500	82.500

5.2 Interference In Internal Gears

Three different types of interference can occur with internal gears:

(a) Involute Interference
(b) Trochoid Interference
(c) Trimming Interference

(a) Involute Interference

This occurs between the dedendum of the external gear and the addendum of the internal gear. It is prevalent when the number of teeth of the external gear is small. Involute interference can be avoided by the conditions cited below:

$$\frac{z_1}{z_2} \geq 1 - \frac{\tan\alpha_{a2}}{\tan\alpha_w} \qquad \textbf{(5-2)}$$

where α_{a2} is the pressure angle seen at a tip of the internal gear tooth.

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \qquad \textbf{(5-3)}$$

and α_w is working pressure angle:

$$\alpha_w = \cos^{-1}\left[\frac{(z_2 - z_1)m\cos\alpha}{2a_x}\right] \qquad \textbf{(5-4)}$$

Equation (5-3) is true only if the outside diameter of the internal gear is bigger than the base circle:

$$d_{a2} \geq d_{b2} \qquad \textbf{(5-5)}$$

Table 5-2 The Calculation of Shifted Internal Gear and External Gear (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		13.1683	
2	Center Distance Increment Factor	y	$\dfrac{a_x}{m} - \dfrac{z_2 - z_1}{2}$	0.38943	
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\dfrac{(z_2 - z_1)\cos\alpha}{2y + z_2 - z_1}\right]$	31.0937°	
4	Difference of Coefficients of Profile Shift	$x_2 - x_1$	$\dfrac{(z_2 - z_1)(\text{inv}\alpha_w - \text{inv}\alpha)}{2\tan\alpha}$	0.5	
5	Coefficient of Profile Shift	x_1, x_2		0	0.5

For a standard internal gear, where α = 20°, **Equation (5-5)** is valid only if the number of teeth is $z_2 > 34$.

(b) Trochoid Interference

This refers to an interference occurring at the addendum of the external gear and the dedendum of the internal gear during recess tooth action. It tends to happen when the difference between the numbers of teeth of the two gears is small. **Equation (5-6)** presents the condition for avoiding trochoidal interference.

$$\theta_1 \frac{z_1}{z_2} + \mathrm{inv}\alpha_w - \mathrm{inv}\alpha_{a2} \geq \theta_2 \qquad \textbf{(5-6)}$$

Here

$$\left.\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{r_{a2}{}^2 - r_{a1}{}^2 - a^2}{2ar_{a1}}\right) + \mathrm{inv}\,\alpha_{a1} - \mathrm{inv}\alpha_w \\ \theta_2 &= \cos^{-1}\left(\frac{a^2 + r_{a2}{}^2 - r_{a1}{}^2}{2ar_{a2}}\right) \end{aligned}\right\} \qquad \textbf{(5-7)}$$

where α_{a1} is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \qquad \textbf{(5-8)}$$

In the meshing of an external gear and a standard internal gear α = 20°, trochoid interference is avoided if the difference of the number of teeth, $z_1 - z_2$, is larger than 9.

(c) Trimming Interference

This occurs in the radial direction in that it prevents pulling the gears apart. Thus, the mesh must be assembled by sliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are very close. **Equation (5-9)** indicates how to prevent this type of interference.

$$\theta_1 + \mathrm{inv}\alpha_{a1} - \mathrm{inv}\alpha_w \geq \frac{z_2}{z_1}(\theta_2 + \mathrm{inv}\alpha_{a2} - \mathrm{inv}\alpha_w) \qquad \textbf{(5-9)}$$

Here

$$\left.\begin{aligned} \theta_1 &= \sin^{-1}\sqrt{\frac{1 - (\cos\alpha_{a1}/\cos\alpha_{a2})^2}{1 - (z_1/z_2)^2}} \\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos\alpha_{a2}/\cos\alpha_{a1})^2 - 1}{(z_2/z_1)^2 - 1}} \end{aligned}\right\} \qquad \textbf{(5-10)}$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen, there is danger of breaking the tooling. **Table 5-3a** shows the limit for the pinion cutter to prevent trimming interference when cutting a standard internal gear, with pressure angle 20°, and no profile shift, i.e., $x_c = 0$.

Table 5-3a The Limit to Prevent an Internal Gear from Trimming Interference (α = 20°, $x_c = x_2 = 0$)

z_c	15	16	17	18	19	20	21	22	24	25	27
z_2	34	34	35	36	37	38	39	40	42	43	45
z_c	28	30	31	32	33	34	35	38	40	42	
z_2	46	48	49	50	51	52	53	56	58	60	
z_c	44	48	50	56	60	64	66	80	96	100	
z_2	62	66	68	74	78	82	84	98	114	118	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 22 (z_c = 15 to 22). **Table 5-3b** shows the limit for a profile shifted pinion cutter to prevent trimming interference while cutting a standard internal gear. The correction, x_c, is the magnitude of shift which was assumed to be: $x_c = 0.0075\, z_c + 0.05$.

Table 5-3b The Limit to Prevent an Internal Gear from Trimming Interference (α = 20°, x_2 = 0)

z_c	15	16	17	18	19	20	21	22	24	25	27
x_c	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525
z_2	36	38	39	40	41	42	43	45	47	48	50
z_c	28	30	31	32	33	34	35	38	40	42	
x_c	0.26	0.275	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365	
z_2	52	54	55	56	58	59	60	64	66	68	
z_c	44	48	50	56	60	64	66	80	96	100	
x_c	0.38	0.41	0.425	0.47	0.5	0.53	0.545	0.65	0.77	0.8	
z_2	71	76	78	86	90	95	98	115	136	141	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 19 (z_c = 15 to 19).

5.3 Internal Gear With Small Differences In Numbers Of Teeth

In the meshing of an internal gear and an external gear, if the difference in numbers of teeth of two gears is quite small, a profile shifted gear could prevent the interference. **Table 5-4** is an example of how to prevent interference under the conditions of z_2 = 50 and the difference of numbers of teeth of two gears ranges from 1 to 8.

Table 5-4 The Meshing of Internal and External Gears of Small Difference of Numbers of Teeth (m = 1, α = 20°)

z_1	49	48	47	46	45	44	43	42
x_1	0							
z_2	50							
x_2	1.00	0.60	0.40	0.30	0.20	0.11	0.06	0.01
α_w	61.0605°	46.0324°	37.4155°	32.4521°	28.2019°	24.5356°	22.3755°	20.3854°
a	0.971	1.354	1.775	2.227	2.666	3.099	3.557	4.010
ε	1.105	1.512	1.726	1.835	1.933	2.014	2.053	2.088

All combinations above will not cause involute interference or trochoid interference, but trimming interference is still there. In order to assemble successfully, the external gear should be assembled by inserting in the axial direction.

A profile shifted internal gear and external gear, in which the difference of numbers of teeth is small, belong to the field of hypocyclic mechanism, which can produce a large reduction ratio in one step, such as 1/100.

$$\text{Speed Ratio} = \frac{z_2 - z_1}{z_1} \qquad \textbf{(5-11)}$$

In **Figure 5-2** the gear train has a difference of numbers of teeth of only 1; $z_1 = 30$ and $z_2 = 31$. This results in a reduction ratio of 1/30.

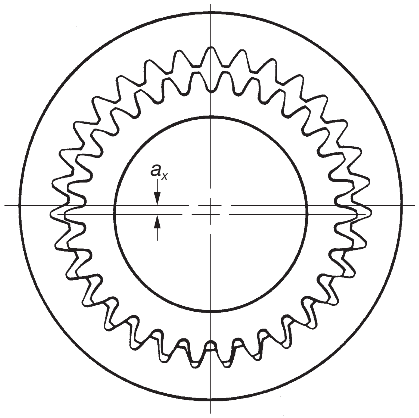


Fig. 5-2 The Meshing of Internal Gear and External Gear in which the Numbers of Teeth Difference is 1
($z_2 - z_1 = 1$)

SECTION 6 HELICAL GEARS

The helical gear differs from the spur gear in that its teeth are twisted along a helical path in the axial direction. It resembles the spur gear in the plane of rotation, but in the axial direction it is as if there were a series of staggered spur gears. See **Figure 6-1**. This design brings forth a number of different features relative to the spur gear, two of the most important being as follows:

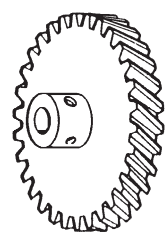


Fig. 6-1 Helical Gear

1. Tooth strength is improved because of the elongated helical wraparound tooth base support.
2. Contact ratio is increased due to the axial tooth overlap. Helical gears thus tend to have greater load carrying capacity than spur gears of the same size. Spur gears, on the other hand, have a somewhat higher efficiency.

Helical gears are used in two forms:

1. Parallel shaft applications, which is the largest usage.
2. Crossed-helicals (also called spiral or screw gears) for connecting skew shafts, usually at right angles.

6.1 Generation Of The Helical Tooth

The helical tooth form is involute in the plane of rotation and can be developed in a manner similar to that of the spur gear. However, unlike the spur gear which can be viewed essentially as two dimensional, the helical gear must be portrayed in three dimensions to show changing axial features.

Referring to **Figure 6-2,** there is a base cylinder from which a taut plane is unwrapped, analogous to the unwinding taut string of the spur gear in **Figure 2-2**. On the plane there is a straight line AB, which when wrapped on the base cylinder has a helical trace A_oB_o. As the taut plane is unwrapped, any point on the line AB can be visualized as tracing an involute from the base cylinder. Thus, there is an infinite series of involutes generated by line AB, all alike, but displaced in phase along a helix on the base cylinder.

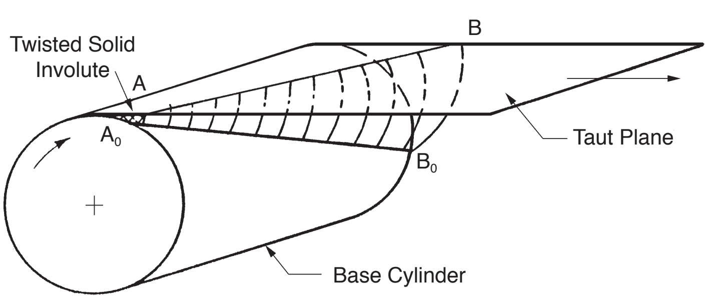


Fig. 6-2 Generation of the Helical Tooth Profile

Again, a concept analogous to the spur gear tooth development is to imagine the taut plane being wound from one base cylinder on to another as the base cylinders rotate in opposite directions. The result is the generation of a pair of conjugate helical involutes. If a reverse direction of rotation is assumed and a second tangent plane is arranged so that it crosses the first, a complete involute helicoid tooth is formed.

6.2 Fundamentals Of Helical Teeth

In the plane of rotation, the helical gear tooth is involute and all of the relationships governing spur gears apply to the helical. However, the axial twist of the teeth introduces a helix angle. Since the helix angle varies from the base of the tooth to the outside radius, the helix angle β is defined as the angle between the tangent to the helicoidal tooth at the intersection of the pitch cylinder and the tooth profile, and an element of the pitch cylinder. See **Figure 6-3**.

The direction of the helical twist is designated as either left or right. The direction is defined by the right-hand rule.

For helical gears, there are two related pitches – one in the plane of rotation and the other in a plane normal to the tooth. In addition, there is an axial pitch.

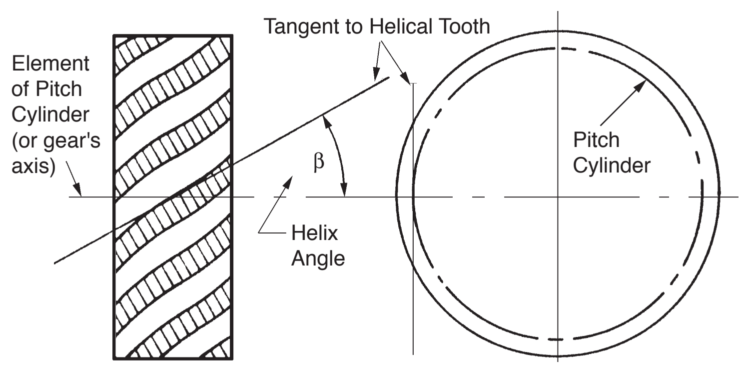


Fig. 6-3 Definition of Helix Angle

Referring to **Figure 6-4**, the two circular pitches are defined and related as follows:

$$p_n = p_t \cos\beta = \text{normal circular pitch} \quad \textbf{(6-1)}$$

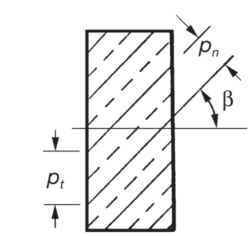


Fig. 6-4 Relationship of Circular Pitches

The normal circular pitch is less than the transverse radial pitch, p_t, in the plane of rotation; the ratio between the two being equal to the cosine of the helix angle.

Consistent with this, the normal module is less than the transverse (radial) module.

The axial pitch of a helical gear, p_x, is the distance between corresponding points of adjacent teeth measured parallel to the gear's axis – see **Figure 6-5**. Axial pitch is related to

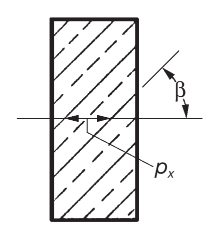


Fig. 6-5 Axial Pitch of a Helical Gear

circular pitch by the expressions:

$$p_x = p_t \cot\beta = \frac{p_n}{\sin\beta} = \text{axial pitch} \tag{6-2}$$

A helical gear such as shown in **Figure 6-6** is a cylindrical gear in which the teeth flank are helicoid. The helix angle in standard pitch circle cylinder is β, and the displacement of one rotation is the lead, *L*.

The tooth profile of a helical gear is an involute curve from an axial

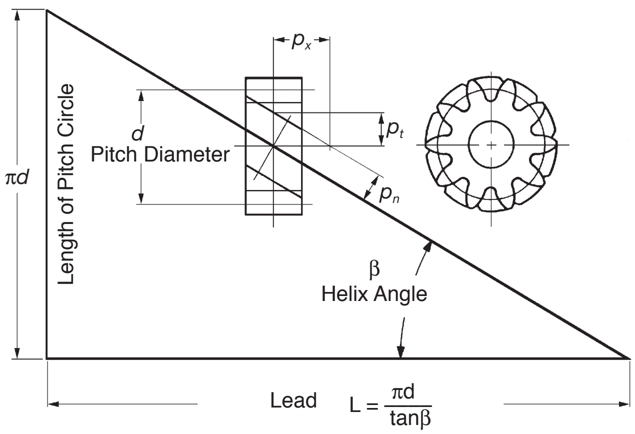


Fig. 6-6 Fundamental Relationship of a Helical Gear (Right-Hand)

view, or in the plane perpendicular to the axis. The helical gear has two kinds of tooth profiles – one is based on a normal system, the other is based on an axial system.

Circular pitch measured perpendicular to teeth is called normal circular pitch, p_n. And p_n divided by π is then a normal module, m_n.

$$m_n = \frac{p_n}{\pi} \tag{6-3}$$

The tooth profile of a helical gear with applied normal module, m_n, and normal pressure angle α_n belongs to a normal system.

In the axial view, the circular pitch on the standard pitch circle is called the radial circular pitch, p_t. And p_t divided by π is the radial module, m_t.

$$m_t = \frac{p_t}{\pi} \tag{6-4}$$

6.3 Equivalent Spur Gear

The true involute pitch and involute geometry of a helical gear is in the plane of rotation. However, in the normal plane, looking at one tooth, there is a resemblance to an involute tooth of a pitch corresponding to the normal pitch. However, the shape of the tooth corresponds to a spur gear of a larger number of teeth, the exact value depending on the magnitude of the helix angle.

The geometric basis of deriving the number of teeth in this equivalent tooth form spur gear is given in **Figure 6-7**. The result of the transposed geometry is an equivalent number of teeth, given as:

$$z_v = \frac{z}{\cos^3\beta} \tag{6-5}$$

This equivalent number is also called a virtual number because this spur gear is imaginary. The value of this number is used in determining helical tooth strength.

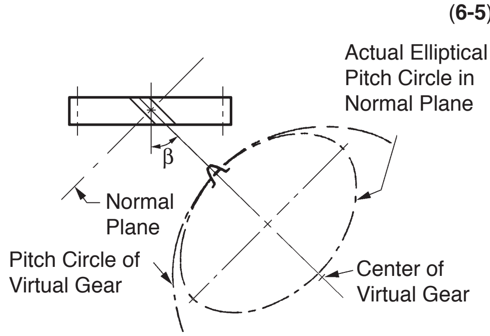


Fig. 6-7 Geometry of Helical Gear's Virtual Number of Teeth

6.4 Helical Gear Pressure Angle

Although, strictly speaking, pressure angle exists only for a gear pair, a nominal pressure angle can be considered for an individual gear. For the helical gear there is a normal pressure, α_n, angle as well as the usual pressure angle in the plane of rotation, α. **Figure 6-8** shows their relationship, which is expressed as:

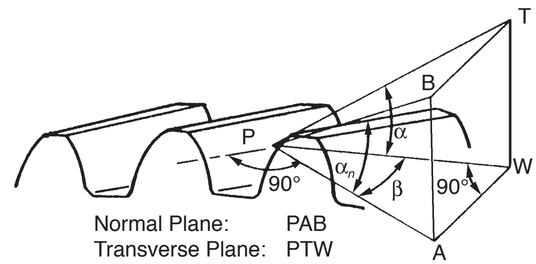


Fig. 6-8 Geometry of Two Pressure Angles

$$\tan\alpha = \frac{\tan\alpha_n}{\cos\beta} \tag{6-6}$$

6.5 Importance Of Normal Plane Geometry

Because of the nature of tooth generation with a rack-type hob, a single tool can generate helical gears at all helix angles as well as spur gears. However, this means the normal pitch is the common denominator, and usually is taken as a standard value. Since the true involute features are in the transverse plane, they will differ from the standard normal values. Hence, there is a real need for relating parameters in the two reference planes.

6.6 Helical Tooth Proportions

These follow the same standards as those for spur gears. Addendum, dedendum, whole depth and clearance are the same regardless of whether measured in the plane of rotation or the normal plane. Pressure angle and pitch are usually specified as standard values in the normal plane, but there are times when they are specified as standard in the transverse plane.

6.7 Parallel Shaft Helical Gear Meshes

Fundamental information for the design of gear meshes is as follows:

Helix angle – Both gears of a meshed pair must have the same helix angle. However, the helix direction must be opposite; i.e., a left-hand mates with a right-hand helix.

Pitch diameter – This is given by the same expression as for spur gears, but if the normal module is involved it is a function of the helix angle. The expressions are:

$$d = z\, m_t = \frac{z}{m_n \cos\beta} \tag{6-7}$$

Center distance – Utilizing **Equation (6-7)**, the center distance of a helical gear mesh is:

$$a = \frac{z_1 + z_2}{2\, m_n \cos\beta} \tag{6-8}$$

Note that for standard parameters in the normal plane, the center distance will not be a standard value compared to standard spur gears. Further, by manipulating the helix angle, β, the center distance can be adjusted over a wide range of values. Conversely, it is possible:

1. to compensate for significant center distance changes (or errors) without changing the speed ratio between parallel geared shafts; and
2. to alter the speed ratio between parallel geared shafts, without changing the center distance, by manipulating the helix angle along with the numbers of teeth.

6.8 Helical Gear Contact Ratio

The contact ratio of helical gears is enhanced by the axial overlap of the teeth. Thus, the contact ratio is the sum of the transverse contact ratio, calculated in the same manner as for spur gears, and a term involving the axial pitch.

$$\left.\begin{array}{l}(\varepsilon)_{total} = (\varepsilon)_{trans} + (\varepsilon)_{axial} \\ \text{or} \\ \varepsilon_r = \varepsilon_\alpha + \varepsilon_\beta\end{array}\right\} \quad (6\text{-}9)$$

Details of contact ratio of helical gearing are given later in a general coverage of the subject; see **SECTION 11.1**.

6.9 Design Considerations

6.9.1 Involute Interference

Helical gears cut with standard normal pressure angles can have considerably higher pressure angles in the plane of rotation – see **Equation (6-6)** – depending on the helix angle. Therefore, the minimum number of teeth without undercutting can be significantly reduced, and helical gears having very low numbers of teeth without undercutting are feasible.

6.9.2 Normal Vs. Radial Module (Pitch)

In the normal system, helical gears can be cut by the same gear hob if module m_n and pressure angle α_n are constant, no matter what the value of helix angle β.

It is not that simple in the radial system. The gear hob design must be altered in accordance with the changing of helix angle β, even when the module m_t and the pressure angle α_t are the same.

Obviously, the manufacturing of helical gears is easier with the normal system than with the radial system in the plane perpendicular to the axis.

6.10 Helical Gear Calculations

6.10.1 Normal System Helical Gear

In the normal system, the calculation of a profile shifted helical gear, the working pitch diameter d_w and working pressure angle α_{wt} in the axial system is done per **Equations (6-10)**. That is because meshing of the helical gears in the axial direction is just like spur gears and the calculation is similar.

$$\left.\begin{array}{l} d_{w1} = 2a_x \dfrac{z_1}{z_1 + z_2} \\ d_{w2} = 2a_x \dfrac{z_2}{z_1 + z_2} \\ \alpha_{wt} = \cos^{-1}\left(\dfrac{d_{b1} + d_{b2}}{2a_x}\right) \end{array}\right\} \quad (6\text{-}10)$$

Table 6-1 shows the calculation of profile shifted helical gears in the normal system. If normal coefficients of profile shift x_{n1}, x_{n2} are zero, they become standard gears.

If center distance, a_x, is given, the normal coefficient of profile shift x_{n1} and x_{n2} can be calculated from **Table 6-2**. These are the inverse equations from items 4 to 10 of **Table 6-1**.

Table 6-1 The Calculation of a Profile Shifted Helical Gear in the Normal System (1)

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	Helix Angle	β		30°	
4	Number of Teeth & Helical Hand	z_1, z_2		12 (L)	60 (R)
5	Radial Pressure Angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	22.79588°	
6	Normal Coefficient of Profile Shift	x_{n1}, x_{n2}		0.09809	0
7	Involute Function α_{wt}	$\text{inv}\ \alpha_{wt}$	$2\tan\alpha_n\left(\dfrac{x_{n1} + x_{n2}}{z_1 + z_2}\right) + \text{inv}\alpha_t$	0.023405	
8	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	23.1126°	
9	Center Distance Increment Factor	y	$\dfrac{z_1 + z_2}{2\cos\beta}\left(\dfrac{\cos\alpha_t}{\cos\alpha_{wt}} - 1\right)$	0.09744	
10	Center Distance	a_x	$\left(\dfrac{z_1 + z_2}{2\cos\beta} + y\right)m_n$	125.000	
11	Standard Pitch Diameter	d	$\dfrac{zm_n}{\cos\beta}$	41.569	207.846
12	Base Diameter	d_b	$d\cos\alpha_t$	38.322	191.611
13	Working Pitch Diameter	h_{a1}	$\dfrac{d_b}{\cos\alpha_{wt}}$	41.667	208.333
14	Addendum	h_{a2}	$(1 + y - x_{n2})\ m_n$ $(1 + y - x_{n1})\ m_n$	3.292	2.998
15	Whole Depth	h	$[2.25 + y - (x_{n1} + x_{n2})]m_n$	6.748	
16	Outside Diameter	d_a	$d + 2\ h_a$	48.153	213.842
17	Root Diameter	d_f	$d_a - 2\ h$	34.657	200.346

Table 6-2 The Calculations of a Profile Shifted Helical Gear in the Normal System (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		125	
2	Center Distance Increment Factor	y	$\dfrac{a_x}{m_n} - \dfrac{z_1 + z_2}{2\cos\beta}$	0.097447	
3	Radial Working Pressure Angle	α_{wt}	$\cos^{-1}\left[\dfrac{(z_1 + z_2)\cos\alpha_t}{(z_1 + z_2) + 2y\cos\beta}\right]$	23.1126°	
4	Sum of Coefficient of Profile Shift	$x_{n1} + x_{n2}$	$\dfrac{(z_1 + z_2)(\text{inv}\ \alpha_{wt} - \text{inv}\ \alpha_t)}{2\tan\alpha_n}$	0.09809	
5	Normal Coefficient of Profile Shift	x_{n1}, x_{n2}		0.09809	0

The transformation from a normal system to a radial system is accomplished by the following equations:

$$\left.\begin{aligned} x_t &= x_n \cos\beta \\ m_t &= \frac{m_n}{\cos\beta} \\ \alpha_t &= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \end{aligned}\right\} \qquad (\mathbf{6\text{-}11})$$

6.10.2 Radial System Helical Gear

Table 6-3 shows the calculation of profile shifted helical gears in a radial system. They become standard if $x_{t1} = x_{t2} = 0$.

Table 6-4 presents the inverse calculation of items 5 to 9 of **Table 6-3**.

The transformation from a radial to a normal system is described by the following equations:

$$\left.\begin{aligned} x_n &= \frac{x_t}{\cos\beta} \\ m_n &= m_t \cos\beta \\ \alpha_n &= \tan^{-1}(\tan\alpha_t \cos\beta) \end{aligned}\right\} \qquad (\mathbf{6\text{-}12})$$

6.10.3 Sunderland Double Helical Gear

A representative application of radial system is a double helical gear, or herringbone gear, made with the Sunderland machine. The radial pressure angle, α_t, and helix angle, β, are specified as 20° and 22.5°, respectively. The only differences from the radial system equations of **Table 6-3** are those for addendum and whole depth. **Table 6-5** presents equations for a Sunderland gear.

6.10.4 Helical Rack

Viewed in the normal direction, the meshing of a helical rack and gear is the same as a spur gear and rack. **Table 6-6** presents the calculation examples for a mated helical rack with normal module and normal pressure angle standard values. Similarly, **Table 6-7** presents examples for a helical rack in the radial system (i.e., perpendicular to gear axis).

Table 6-3 The Calculation of a Profile Shifted Helical Gear in the Radial System (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Radial Module	m_t		3	
2	Radial Pressure Angle	α_t		20°	
3	Helix Angle	β		30°	
4	Number of Teeth & Helical Hand	z_1, z_2		12 (L)	60 (R)
5	Radial Coefficient of Profile Shift	x_{t1}, x_{t2}		0.34462	0
6	Involute Function α_{wt}	inv α_{wt}	$2\tan\alpha_t\left(\frac{x_{t1}+x_{t2}}{z_1+z_2}\right) + \text{inv}\alpha_t$	0.0183886	
7	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	21.3975°	
8	Center Distance Increment Factor	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}} - 1\right)$	0.33333	
9	Center Distance	a_x	$\left(\frac{z_1+z_2}{2} + y\right)m_t$	109.0000	
10	Standard Pitch Diameter	d	zm_t	36.000	180.000
11	Base Diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(1 + y - x_{t2})\,m_t$ $(1 + y - x_{t1})\,m_t$	4.000	2.966
14	Whole Depth	h	$[2.25 + y - (x_{t1} + x_{t2})]m_t$	6.716	
15	Outside Diameter	d_a	$d + 2h_a$	44.000	185.932
16	Root Diameter	d_f	$d_a - 2h$	30.568	172.500

Table 6-4 The Calculation of a Shifted Helical Gear in the Radial System (2)

No.	Item	Symbol	Formula		
1	Center Distance	a_x		109	
2	Center Distance Increment Factor	y	$\frac{a_x}{m_t} - \frac{z_1+z_2}{2}$	0.33333	
3	Radial Working Pressure Angle	α_{wt}	$\cos^{-1}\left[\frac{(z_1+z_2)\cos\alpha_t}{(z_1+z_2)+2y}\right]$	21.39752°	
4	Sum of Coefficient of Profile Shift	$x_{t1}+x_{t2}$	$\frac{(z_1+z_2)(\text{inv}\,\alpha_{wt} - \text{inv}\,\alpha_t)}{2\tan\alpha_n}$	0.34462	
5	Normal Coefficient of Profile Shift	x_{t1}, x_{t2}		0.34462	0

Table 6-5 The Calculation of a Double Helical Gear of SUNDERLAND Tooth Profile

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Radial Module	m_t		3	
2	Radial Pressure Angle	α_t		20°	
3	Helix Angle	β		22.5°	
4	Number of Teeth	z_1, z_2		12	60
5	Radial Coefficient of Profile Shift	x_{t1}, x_{t2}		0.34462	0
6	Involute Function α_{wt}	inv α_{wt}	$2\tan\alpha_t \left(\frac{x_{t1}+x_{t2}}{z_1+z_2}\right) + \text{inv}\alpha_t$	0.0183886	
7	Radial Working Pressure Angle	α_{wt}	Find from Involute Function Table	21.3975°	
8	Center Distance Increment Factor	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}}-1\right)$	0.33333	
9	Center Distance	a_x	$\left(\frac{z_1+z_2}{2}+y\right)m_t$	109.0000	
10	Standard Pitch Diameter	d	zm_t	36.000	180.000
11	Base Diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_{wt}}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(0.8796+y-x_{t2})\,m_t$ $(0.8796+y-x_{t1})\,m_t$	3.639	2.605
14	Whole Depth	h	$[1.8849+y-(x_{t1}+x_{t2})]\,m_t$	5.621	
15	Outside Diameter	d_a	$d+2h_a$	43.278	185.210
16	Root Diameter	d_f	d_a-2h	32.036	173.968

Table 6-6 The Calculation of a Helical Rack in the Normal System

No.	Item	Symbol	Formula	Example	
				Gear	Rack
1	Normal Module	m_n		2.5	
2	Normal Pressure Angle	α_n		20°	
3	Helix Angle	β		10° 57' 49"	
4	Number of Teeth & Helical Hand	z		20 (R)	– (L)
5	Normal Coefficient of Profile Shift	x_n		0	–
6	Pitch Line Height	H		–	27.5
7	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	20.34160°	
8	Mounting Distance	a_x	$\frac{zm_n}{2\cos\beta}+H+x_n m_n$	52.965	
9	Pitch Diameter	d	$\frac{zm_n}{\cos\beta}$	50.92956	–
10	Base Diameter	d_b	$d\cos\alpha_t$	47.75343	
11	Addendum	h_a	$m_n(1+x_n)$	2.500	2.500
12	Whole Depth	h	$2.25m_n$	5.625	
13	Outside Diameter	d_a	$d+2h_a$	55.929	–
14	Root Diameter	d_f	d_a-2h	44.679	

Table 6-7 The Calculation of a Helical Rack in the Radial System

No.	Item	Symbol	Formula	Example	
				Gear	Rack
1	Radial Module	m_t		2.5	
2	Radial Pressure Angle	α_t		20°	
3	Helix Angle	β		10° 57' 49"	
4	Number of Teeth & Helical Hand	z		20 (R)	– (L)
5	Radial Coefficient of Profile Shift	x_t		0	–
6	Pitch Line Height	H		–	27.5
7	Mounting Distance	a_x	$\frac{zm_t}{2}+H+x_t m_t$	52.500	
8	Pitch Diameter	d	zm_t	50.000	–
9	Base Diameter	d_b	$d\cos\alpha_t$	46.98463	
10	Addendum	h_a	$m_t(1+x_t)$	2.500	2.500
11	Whole Depth	h	$2.25m_t$	5.625	
12	Outside Diameter	d_a	$d+2h_a$	55.000	–
13	Root Diameter	d_f	d_a-2h	43.750	

The formulas of a standard helical rack are similar to those of **Table 6-6** with only the normal coefficient of profile shift $x_n = 0$. To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, l, for one rotation of the mating gear is the product of the radial pitch, p_t, and number of teeth.

$$l = \frac{\pi m_n}{\cos\beta} z = p_t z \qquad \textbf{(6-13)}$$

According to the equations of **Table 6-7,** let radial pitch p_t = 8 mm and displacement l = 160 mm. The radial pitch and the displacement could be modified into integers, if the helix angle were chosen properly.

In the axial system, the linear displacement of the helical rack, l, for one turn of the helical gear equals the integral multiple of radial pitch.

$$l = \pi z m_t \qquad \textbf{(6-14)}$$

SECTION 7 SCREW GEAR OR CROSSED HELICAL GEAR MESHES

These helical gears are also known as spiral gears. They are true helical gears and only differ in their application for interconnecting skew shafts, such as in **Figure 7-1**. Screw gears can be designed to connect shafts at any angle, but in most applications the shafts are at right angles.

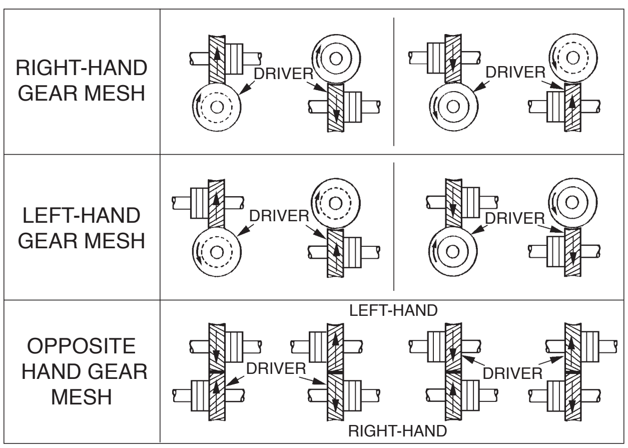


Fig. 7-1 Types of Helical Gear Meshes

NOTES:
1. Helical gears of the same hand operate at right angles.
2. Helical gears of opposite hand operate on parallel shafts.
3. Bearing location indicates the direction of thrust.

7.1 Features

7.1.1 Helix Angle And Hands

The helix angles need not be the same. However, their sum must equal the shaft angle:

$$\beta_1 + \beta_2 = \Sigma \qquad \textbf{(7-1)}$$

where β_1 and β_2 are the respective helix angles of the two gears, and Σ is the shaft angle (the acute angle between the two shafts when viewed in a direction paralleling a common perpendicular between the shafts).

Except for very small shaft angles, the helix hands are the same.

7.1.2 Module

Because of the possibility of different helix angles for the gear pair, the radial modules may not be the same. However, the normal modules must always be identical.

7.1.3 Center Distance

The pitch diameter of a crossed-helical gear is given by **Equation (6-7)**, and the center distance becomes:

$$a = \frac{m_n}{2}\left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2}\right) \qquad \textbf{(7-2)}$$

Again, it is possible to adjust the center distance by manipulating the helix angle. However, helix angles of both gears must be altered consistently in accordance with **Equation (7-1)**.

7.1.4 Velocity Ratio

Unlike spur and parallel shaft helical meshes, the velocity ratio (gear ratio) cannot be determined from the ratio of pitch diameters, since these can be altered by juggling of helix angles. The speed ratio can be determined only from the number of teeth, as follows:

$$\text{velocity ratio} = i = \frac{z_1}{z_2} \qquad \textbf{(7-3)}$$

or, if pitch diameters are introduced, the relationship is:

$$i = \frac{z_1 \cos\beta_2}{z_2 \cos\beta_1} \qquad \textbf{(7-4)}$$

7.2 Screw Gear Calculations

Two screw gears can only mesh together under the conditions that normal modules, m_{n1}, and, m_{n2}, and normal pressure angles, α_{n1}, α_{n2}, are the same. Let a pair of screw gears have the shaft angle Σ and helical angles β_1 and β_2:

$$\left.\begin{aligned} &\text{If they have the same hands, then:} \\ &\quad \Sigma = \beta_1 + \beta_2 \\ &\text{If they have the opposite hands, then:} \\ &\quad \Sigma = \beta_1 - \beta_2, \text{ or } \Sigma = \beta_2 - \beta_1 \end{aligned}\right\} \qquad \textbf{(7-5)}$$

If the screw gears were profile shifted, the meshing would become a little more complex. Let β_{w1}, β_{w2} represent the working pitch cylinder;

$$\left.\begin{aligned} &\text{If they have the same hands, then:} \\ &\quad \Sigma = \beta_{w1} + \beta_{w2} \\ &\text{If they have the opposite hands, then:} \\ &\quad \Sigma = \beta_{w1} - \beta_{w2}, \text{ or } \Sigma = \beta_{w2} - \beta_{w1} \end{aligned}\right\} \qquad \textbf{(7-6)}$$

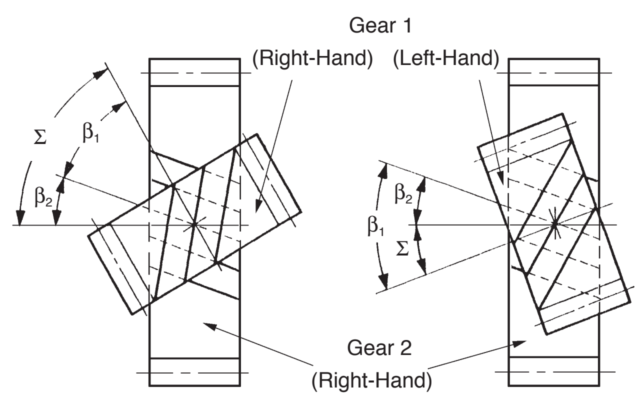


Fig. 7-2 Screw Gears of Nonparallel and Nonintersecting Axes

Table 7-1 presents equations for a profile shifted screw gear pair. When the normal coefficients of profile shift $x_{n1} = x_{n2} = 0$, the equations and calculations are the same as for standard gears.

Standard screw gears have relations as follows:

$$\left.\begin{array}{l} d_{w1} = d_1,\ d_{w2} = d_2 \\ \beta_{w1} = \beta_1,\ \beta_{w2} = \beta_2 \end{array}\right\} \qquad (\mathbf{7\text{-}7})$$

7.3 Axial Thrust Of Helical Gears

In both parallel-shaft and crossed-shaft applications, helical gears develop an axial thrust load. This is a useless force that loads gear teeth and bearings and must accordingly be considered in the housing and bearing design. In some special instrument designs, this thrust load can be utilized to actuate face clutches, provide a friction drag, or other special purpose. The magnitude of the thrust load depends on the helix angle and is given by the expression:

$$W_T = W^t \tan\beta \qquad (\mathbf{7\text{-}8})$$

where

W_T = axial thrust load, and
W^t = transmitted load.

The direction of the thrust load is related to the hand of the gear and the direction of rotation. This is depicted in **Figure 7-1**. When the helix angle is larger than about 20°, the use of double helical gears with opposite hands (**Figure 7-3a**) or herringbone gears (**Figure 7-3b**) is worth considering.

More detail on thrust force of helical gears is presented in **SECTION 16**.

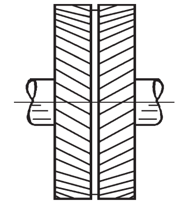


Figure 7-3a

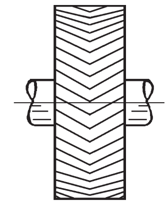


Figure 7-3b

Table 7-1 The Equations for a Screw Gear Pair on Nonparallel and Nonintersecting Axes in the Normal System

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	Helix Angle	β		20°	30°
4	Number of Teeth & Helical Hand	z_1, z_2		15 (R)	24 (L)
5	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3\beta}$	18.0773	36.9504
6	Radial Pressure Angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	21.1728°	22.7959°
7	Normal Coefficient of Profile Shift	x_n		0.4	0.2
8	Involute Function α_{wn}	$\mathrm{inv}\alpha_{wn}$	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_{v1}+z_{v2}}\right)+\mathrm{inv}\alpha_n$	0.0228415	
9	Normal Working Pressure Angle	α_{wn}	Find from Involute Function Table	22.9338°	
10	Radial Working Pressure Angle	α_{wt}	$\tan^{-1}\left(\frac{\tan\alpha_{wn}}{\cos\beta}\right)$	24.2404°	26.0386°
11	Center Distance Increment Factor	y	$\frac{1}{2}(z_{v1}+z_{v2})\left(\frac{\cos\alpha_n}{\cos\alpha_{wn}}-1\right)$	0.55977	
12	Center Distance	a_x	$\left(\frac{z_1}{2\cos\beta_1}+\frac{z_2}{2\cos\beta_2}+y\right)m_n$	67.1925	
13	Pitch Diameter	d	$\frac{zm_n}{\cos\beta}$	47.8880	83.1384
14	Base Diameter	d_b	$d\cos\alpha_t$	44.6553	76.6445
15	Working Pitch Diameter	d_{w1} d_{w2}	$2a_x\frac{d_1}{d_1+d_2}$ $2a_x\frac{d_2}{d_1+d_2}$	49.1155	85.2695
16	Working Helix Angle	β_w	$\tan^{-1}\left(\frac{d_w}{d}\tan\beta\right)$	20.4706°	30.6319°
17	Shaft Angle	Σ	$\beta_{w1}+\beta_{w2}$ or $\beta_{w1}-\beta_{w2}$	51.1025°	
18	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})m_n$ $(1+y-x_{n1})m_n$	4.0793	3.4793
19	Whole Depth	h	$[2.25+y-(x_{n1}+x_{n2})]m_n$	6.6293	
20	Outside Diameter	d_a	$d+2h_a$	56.0466	90.0970
21	Root Diameter	d_f	d_a-2h	42.7880	76.8384

SECTION 8 BEVEL GEARING

For intersecting shafts, bevel gears offer a good means of transmitting motion and power. Most transmissions occur at right angles, **Figure 8-1**, but the shaft angle can be any value. Ratios up to 4:1 are common, although higher ratios are possible as well.

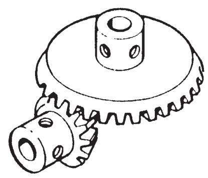
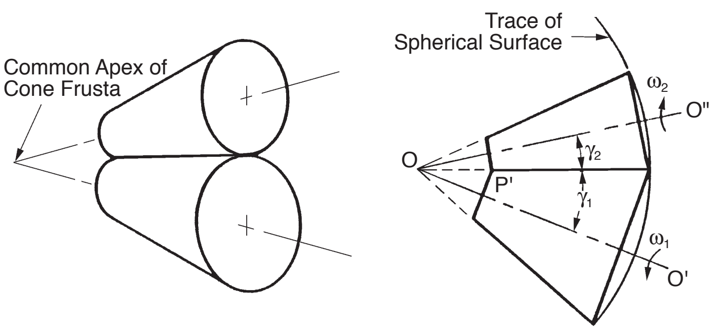


Fig. 8-1 Typical Right Angle Bevel Gear

8.1 Development And Geometry Of Bevel Gears

Bevel gears have tapered elements because they are generated and operate, in theory, on the surface of a sphere. Pitch diameters of mating bevel gears belong to frusta of cones, as shown in **Figure 8-2a**. In the full development on the surface of a sphere, a pair of meshed bevel gears are in conjugate engagement as shown in **Figure 8-2b**.



(a) Pitch Cone Frusta **(b)** Pitch Cones and the Development Sphere

Fig. 8-2 Pitch Cones of Bevel Gears

The crown gear, which is a bevel gear having the largest possible pitch angle (defined in **Figure 8-3**), is analogous to the rack of spur gearing, and is the basic tool for generating bevel gears. However, for practical reasons, the tooth form is not that of a spherical involute, and instead, the crown gear profile assumes a slightly simplified form. Although the deviation from a true spherical involute is minor, it results in a line-of-action having a figure-8 trace in its extreme extension; see **Figure 8-4**. This shape gives rise to the name "octoid" for the tooth form of modern bevel gears.

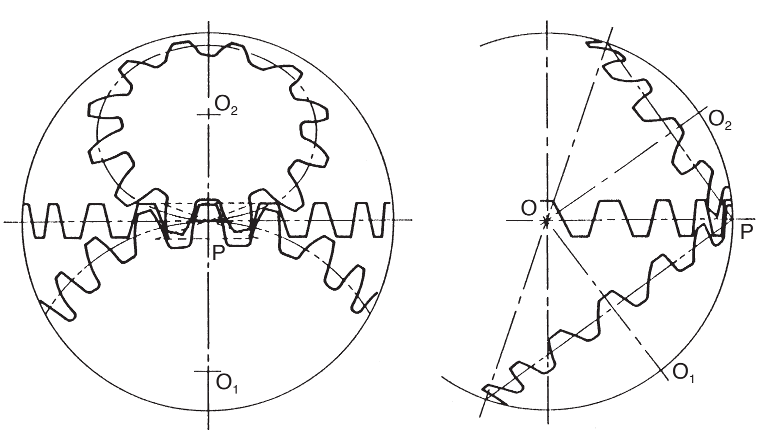


Fig. 8-3 Meshing Bevel Gear Pair with Conjugate Crown Gear

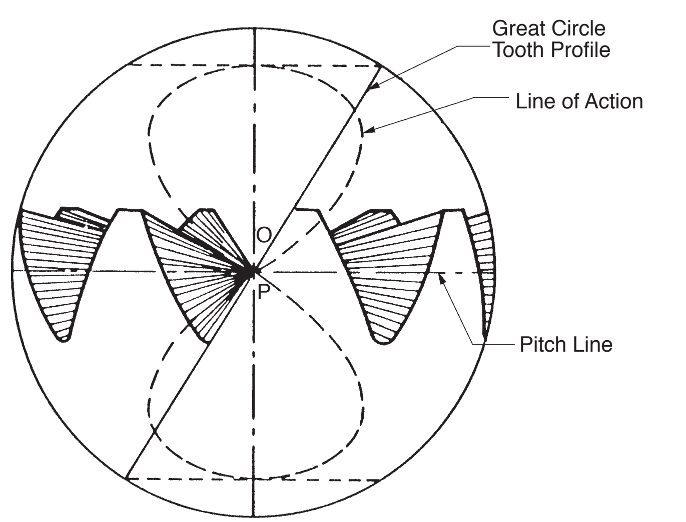


Fig. 8-4 Spherical Basis of Octoid Bevel Crown Gear

8.2 Bevel Gear Tooth Proportions

Bevel gear teeth are proportioned in accordance with the standard system of tooth proportions used for spur gears. However, the pressure angle of all standard design bevel gears is limited to 20°. Pinions with a small number of teeth are enlarged automatically when the design follows the Gleason system.

Since bevel-tooth elements are tapered, tooth dimensions and pitch diameter are referenced to the outer end (heel). Since the narrow end of the teeth (toe) vanishes at the pitch apex (center of reference generating sphere), there is a practical limit to the length (face) of a bevel gear. The geometry and identification of bevel gear parts is given in **Figure 8-5**.

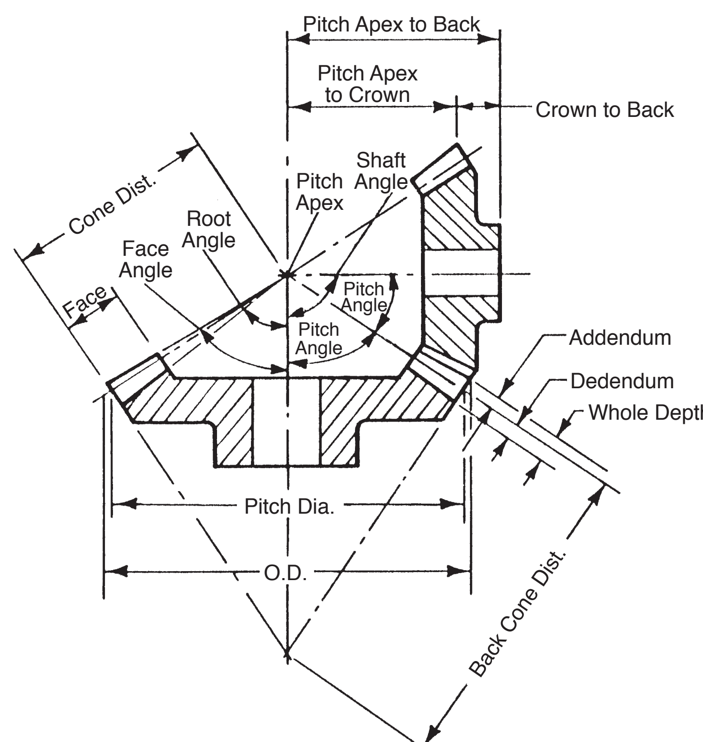


Fig. 8-5 Bevel Gear Pair Design Parameters

8.3 Velocity Ratio

The velocity ratio, i , can be derived from the ratio of several parameters:

$$i = \frac{z_1}{z_2} = \frac{d_1}{d_2} = \frac{\sin\delta_1}{\sin\delta_2} \tag{8-1}$$

where: δ = pitch angle (see **Figure 8-5**)

8.4 Forms Of Bevel Teeth *

In the simplest design, the tooth elements are straight radial, converging at the cone apex. However, it is possible to have the teeth curve along a spiral as they converge on the cone apex, resulting in greater tooth overlap, analogous to the overlapping action of helical teeth. The result is a spiral bevel tooth. In addition, there are other possible variations. One is the zerol bevel, which is a curved tooth having elements that start and end on the same radial line.

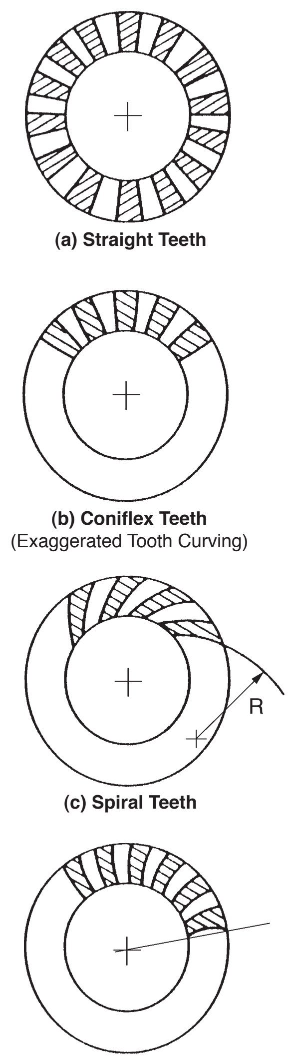
Straight bevel gears come in two variations depending upon the fabrication equipment. All current Gleason straight bevel generators are of the Coniflex form which gives an almost imperceptible convexity to the tooth surfaces. Older machines produce true straight elements. See **Figure 8-6a**.

Straight bevel gears are the simplest and most widely used type of bevel gears for the transmission of power and/or motion between intersecting shafts. Straight bevel gears are recommended:

1. When speeds are less than 300 meters/min (1000 feet/min) – at higher speeds, straight bevel gears may be noisy.
2. When loads are light, or for high static loads when surface wear is not a critical factor.
3. When space, gear weight, and mountings are a premium. This includes planetary gear sets, where space does not permit the inclusion of rolling-element bearings.

Other forms of bevel gearing include the following:

- Coniflex gears (**Figure 8-6b**) are produced by current Gleason straight bevel gear generating machines that crown the sides of the teeth in their lengthwise direction. The teeth, therefore, tolerate small amounts of misalignment in the assembly of the gears and some displacement of the gears under load without concentrating the tooth contact at the ends of the teeth. Thus, for the operating conditions, Coniflex gears are capable of transmitting larger loads than the predecessor Gleason straight bevel gears.
- Spiral bevels (**Figure 8-6c**) have curved oblique teeth which contact each other gradually and smoothly from one end to the other. Imagine cutting a straight bevel into an infinite number of short face width sections, angularly displace one relative to the other, and one has a spiral bevel gear. Well-designed spiral bevels have two or more teeth in contact at all times. The overlapping tooth action transmits motion more smoothly and quietly than with straight bevel gears.



(a) Straight Teeth

(b) Coniflex Teeth
(Exaggerated Tooth Curving)

(c) Spiral Teeth

(d) Zerol Teeth

Fig. 8-6 Forms of Bevel Gear Teeth

- Zerol bevels (**Figure 8-6d**) have curved teeth similar to those of the spiral bevels, but with zero spiral angle at the middle of the face width; and they have little end thrust.

Both spiral and Zerol gears can be cut on the same machines with the same circular face-mill cutters or ground on the same grinding machines. Both are produced with localized tooth contact which can be controlled for length, width, and shape.

Functionally, however, Zerol bevels are similar to the straight bevels and thus carry the same ratings. In fact, Zerols can be used in the place of straight bevels without mounting changes.

Zerol bevels are widely employed in the aircraft industry, where ground-tooth precision gears are generally required. Most hypoid cutting machines can cut spiral bevel, Zerol or hypoid gears.

8.5 Bevel Gear Calculations

Let z_1 and z_2 be pinion and gear tooth numbers; shaft angle Σ; and pitch cone angles δ_1 and δ_2; then:

$$\left.\begin{aligned} \tan\delta_1 &= \frac{\sin\Sigma}{\dfrac{z_2}{z_1} + \cos\Sigma} \\ \tan\delta_2 &= \frac{\sin\Sigma}{\dfrac{z_1}{z_2} + \cos\Sigma} \end{aligned}\right\} \tag{8-2}$$

Generally, shaft angle $\Sigma = 90°$ is most used. Other angles (**Figure 8-7**) are sometimes used. Then, it is called "bevel gear in nonright angle drive". The 90° case is called "bevel gear in right angle drive".

When $\Sigma = 90°$, **Equation (8-2)** becomes:

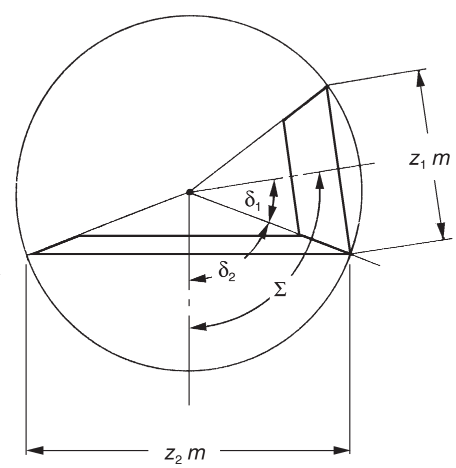


Fig. 8-7 The Pitch Cone Angle of Bevel Gear

$$\left.\begin{aligned} \delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right) \\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right) \end{aligned}\right\} \tag{8-3}$$

Miter gears are bevel gears with $\Sigma = 90°$ and $z_1 = z_2$. Their speed ratio $z_1 / z_2 = 1$. They only change the direction of the shaft, but do not change the speed.

Figure 8-8 depicts the meshing of bevel gears. The meshing must be considered in pairs. It is because the pitch cone angles δ_1 and δ_2 are restricted by the gear ratio z_1 / z_2. In the facial view, which is normal to the contact line of pitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

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8.5.1 Gleason Straight Bevel Gears

The straight bevel gear has straight teeth flanks which are along the surface of the pitch cone from the bottom to the apex. Straight bevel gears can be grouped into the Gleason type and the standard type.

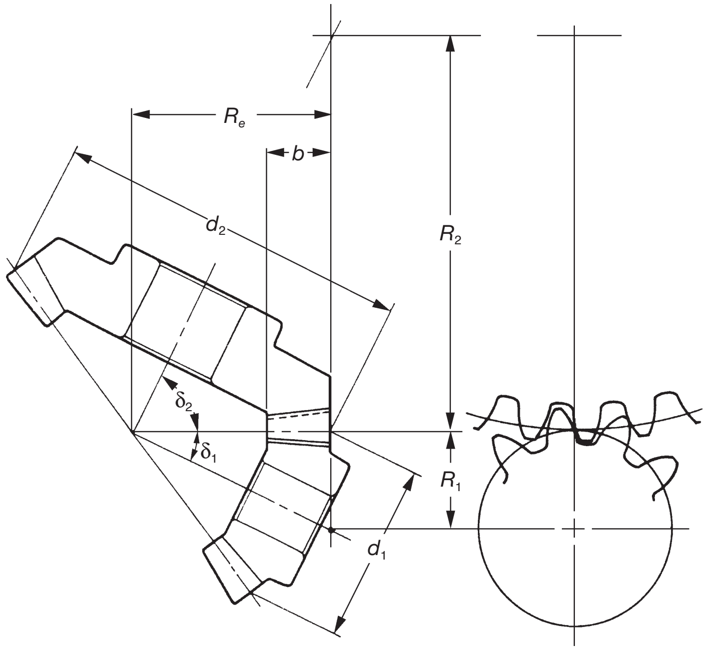


Fig. 8-8 The Meshing of Bevel Gears

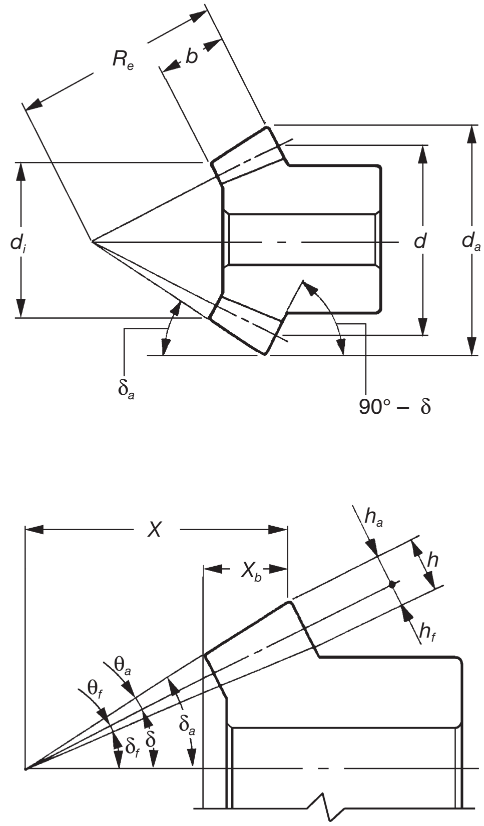


Fig. 8-9 Dimensions and Angles of Bevel Gears

In this section, we discuss the Gleason straight bevel gear. The Gleason Company defined the tooth profile as: whole depth h =2.188m; top clearance c_a = 0.188m; and working depth h_w = 2.000m.

The characteristics are:

- **Design specified profile shifted gears:**

In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distribute the proper strength between the two gears. Miter gears, thus, do not need any shifted tooth profile.

- **The top clearance is designed to be parallel**

The outer cone elements of two paired bevel gears are parallel. That is to ensure that the top clearance along the whole tooth is the same. For the standard bevel gears, top clearance is variable. It is smaller at the toe and bigger at the heel.

Table 8-1 shows the minimum number of teeth to prevent undercut in the Gleason system at the shaft angle $\Sigma = 90°$.

Table 8-2 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensions and angles are shown in **Figure 8-9**. All the equations in **Table 8-2** can also be applied to bevel gears with any shaft angle.

The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a special Gleason "Coniflex" machine. It can successfully eliminate poor tooth wear due to improper mounting and assembly.

The first characteristic of a Gleason straight bevel gear is its profile shifted tooth. From **Figure 8-10**, we can see the positive tooth profile shift in the pinion. The tooth thickness at the root diameter of a Gleason pinion is larger than that of a standard straight bevel gear.

Table 8-1 The Minimum Numbers of Teeth to Prevent Undercut

Pressure Angle	Combination of Numbers of Teeth $\frac{z_1}{z_2}$					
(14.5°)	29 / Over 29	28 / Over 29	27 / Over 31	26 / Over 35	25 / Over 40	24 / Over 57
20°	16 / Over 16	15 / Over 17	14 / Over 20	13 / Over 30	—	—
(25°)	13 / Over 13	—	—	—	—	—

8.5.2. Standard Straight Bevel Gears

A bevel gear with no profile shifted tooth is a standard straight bevel gear. The applicable equations are in **Table 8-3**.

These equations can also be applied to bevel gear sets with other than 90° shaft angle.

8.5.3 Gleason Spiral Bevel Gears

A spiral bevel gear is one with a spiral tooth flank as in **Figure 8-11**. The spiral is generally consistent with the curve of a cutter with the diameter d_c. The spiral angle β is the angle between a generatrix element of the pitch cone and the tooth flank. The spiral angle just at the tooth flank center is called central spiral angle β_m. In practice, spiral angle means central spiral angle.

All equations in **Table 8-6** are dedicated for the manufacturing method of Spread Blade or of Single Side from Gleason. If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the whole depth h = 1.888m; top clearance c_a = 0.188m; and working depth h_w = 1.700m. These Gleason spiral bevel gears belong to a stub gear system. This is applicable to gears with modules $m > 2.1$.

Table 8-4 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle $\Sigma = 90°$ and pressure angle $\alpha_n = 20°$.

If the number of teeth is less than 12, **Table 8-5** is used to determine the gear sizes.

All equations in **Table 8-6** are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear set requires matching of hands; left-hand and right-hand as a pair.

Table 8-2 The Calculations of Straight Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Shaft Angle	Σ		90°	
2	Module	m		3	
3	Pressure Angle	α		20°	
4	Number of Teeth	z_1, z_2		20	40
5	Pitch Diameter	d	zm	60	120
6	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Face Width	b	It should be less than R_e / 3 or 10m	22	
9	Addendum	h_{a1} h_{a2}	$2.000m-h_{a2}$ $0.540m+\dfrac{0.460m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m-h_a$	2.529	4.599
11	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	2.15903°	3.92194°
12	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194°	2.15903°
13	Outer Cone Angle	δ_a	$\delta+\theta_a$	30.48699°	65.59398°
14	Root Cone Angle	δ_f	$\delta-\theta_f$	24.40602°	59.51301°
15	Outside Diameter	d_a	$d+2h_a\cos\delta$	67.2180	121.7575
16	Pitch Apex to Crown	X	$R_e\cos\delta-h_a\sin\delta$	58.1955	28.2425
17	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner Outside Diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

Table 8-3 Calculation of a Standard Straight Bevel Gears

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Shaft Angle	Σ		90°	
2	Module	m		3	
3	Pressure Angle	α		20°	
4	Number of Teeth	z_1, z_2		20	40
5	Pitch Diameter	d	zm	60	120
6	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Face Width	b	It should be less than R_e/3 or 10m	22	
9	Addendum	h_a	1.00 m	3.00	
10	Dedendum	h_f	1.25 m	3.75	
11	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	3.19960°	
12	Addendum Angle	θ_a	$\tan^{-1}(h_a/R_e)$	2.56064°	
13	Outer Cone Angle	δ_a	$\delta+\theta_a$	29.12569°	65.99559°
14	Root Cone Angle	δ_f	$\delta-\theta_f$	23.36545°	60.23535°
15	Outside Diameter	d_a	$d+2h_a\cos\delta$	65.3666	122.6833
16	Pitch Apex to Crown	X	$R_e\cos\delta-h_a\sin\delta$	58.6584	27.3167
17	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.2374	8.9587
18	Inner Outside Diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	43.9292	82.4485

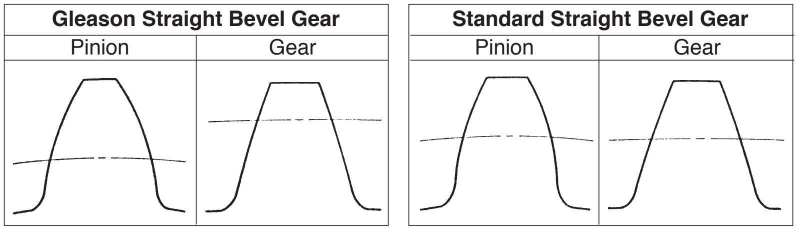


Fig. 8-10 The Tooth Profile of Straight Bevel Gears

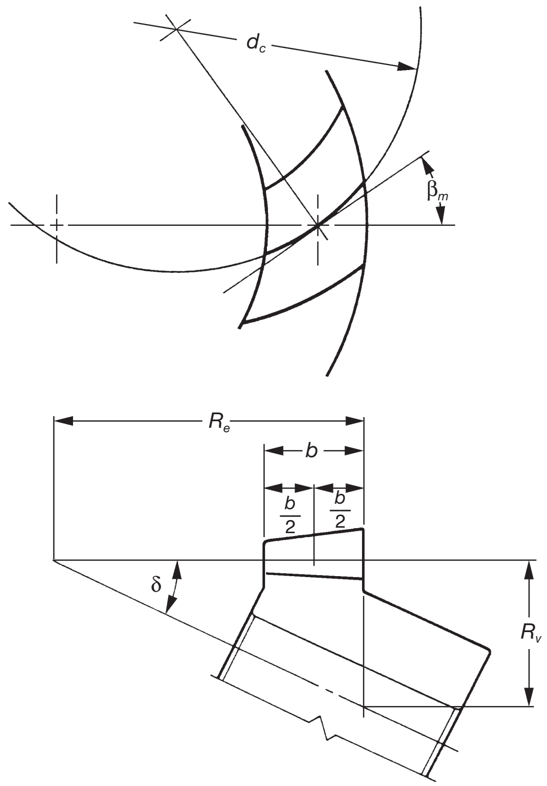


Fig. 8-11 Spiral Bevel Gear (Left-Hand)

Table 8-4 The Minimum Numbers of Teeth to Prevent Undercut $\beta_m = 35°$

Pressure Angle	Combination of Numbers of Teeth $\frac{z_1}{z_2}$					
20°	17 / Over 17	16 / Over 18	15 / Over 19	14 / Over 20	13 / Over 22	12 / Over 26

Table 8-5 Dimensions for Pinions with Numbers of Teeth Less than 12

Number of Teeth in Pinion	z_1	6	7	8	9	10	11
Number of Teeth in Gear	z_2	Over 34	Over 33	Over 32	Over 31	Over 30	Over 29
Working Depth	h_w	1.500	1.560	1.610	1.650	1.680	1.695
Whole Depth	h	1.666	1.733	1.788	1.832	1.865	1.882
Gear Addendum	h_{a2}	0.215	0.270	0.325	0.380	0.435	0.490
Pinion Addendum	h_{a1}	1.285	1.290	1.285	1.270	1.245	1.205
Circular Tooth Thickness of Gear s_2	30	0.911	0.957	0.975	0.997	1.023	1.053
	40	0.803	0.818	0.837	0.860	0.888	0.948
	50	—	0.757	0.777	0.828	0.884	0.946
	60	—	—	0.777	0.828	0.883	0.945
Pressure Angle	α_n	20°					
Spiral Angle	β_m	35°... 40°					
Shaft Angle	Σ	90°					

NOTE: All values in the table are based on m = 1.

8.5.4 Gleason Zerol Spiral Bevel Gears

When the spiral angle $\beta_m = 0$, the bevel gear is called a Zerol bevel gear. The calculation equations of **Table 8-2** for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched. **Figure 8-12** is a left-hand Zerol bevel gear.

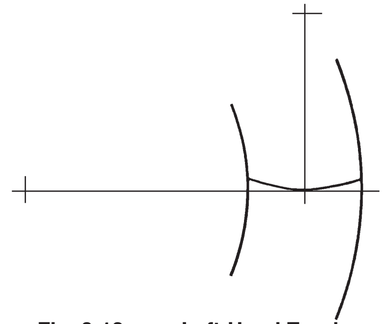


Fig. 8-12 Left-Hand Zerol Bevel Gear

Table 8-6 The Calculations of Spiral Bevel Gears of the Gleason System

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Shaft Angle	Σ		90°	
2	Outside Radial Module	m		3	
3	Normal Pressure Angle	α_n		20°	
4	Spiral Angle	β_m		35°	
5	Number of Teeth and Spiral Hand	z_1, z_2		20 (L)	40 (R)
6	Radial Pressure Angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta_m}\right)$	23.95680	
7	Pitch Diameter	d	zm	60	120
8	Pitch Cone Angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma - \delta_1$	26.56505°	63.43495°
9	Cone Distance	R_e	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
10	Face Width	b	It should be less than $R_e/3$ or $10m$	20	
11	Addendum	h_{a1} h_{a2}	$1.700m - h_{a2}$ $0.460m + \dfrac{0.390m}{\left(\frac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m - h_a$	2.2365	3.9915
13	Dedendum Angle	θ_f	$\tan^{-1}(h_f/R_e)$	1.90952°	3.40519°
14	Addendum Angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.40519°	1.90952°
15	Outer Cone Angle	δ_a	$\delta + \theta_a$	29.97024°	65.34447°
16	Root Cone Angle	δ_f	$\delta - \theta_f$	24.65553°	60.02976°
17	Outside Diameter	d_a	$d + 2h_a\cos\delta$	66.1313	121.4959
18	Pitch Apex to Crown	X	$R_e\cos\delta - h_a\sin\delta$	58.4672	28.5041
19	Axial Face Width	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	17.3563	8.3479
20	Inner Outside Diameter	d_i	$d_a - \dfrac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

SECTION 9 WORM MESH

The worm mesh is another gear type used for connecting skew shafts, usually 90°. See **Figure 9-1**. Worm meshes are characterized by high velocity ratios. Also, they offer the advantage of higher load capacity associated with their line contact in contrast to the point contact of the crossed-helical mesh.

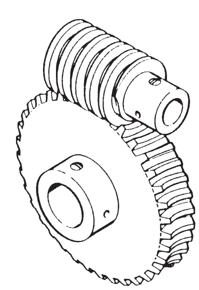


Fig. 9-1 Typical Worm Mesh

9.1 Worm Mesh Geometry

Although the worm tooth form can be of a variety, the most popular is equivalent to a V-type screw thread, as in **Figure 9-1**. The mating worm gear teeth have a helical lead. (**Note:** The name "worm wheel" is often used interchangeably with "worm gear".) A central section of the mesh, taken through the worm's axis and perpendicular to the worm gear's axis, as shown in **Figure 9-2**, reveals a rack-type tooth of the worm, and a curved involute tooth form for the worm gear. However, the involute features are only true for the central section. Sections on either side of the worm axis reveal nonsymmetric and noninvolute tooth profiles. Thus, a worm gear mesh is not a true involute mesh. Also, for conjugate action, the center distance of the mesh must be an exact duplicate of that used in generating the worm gear.

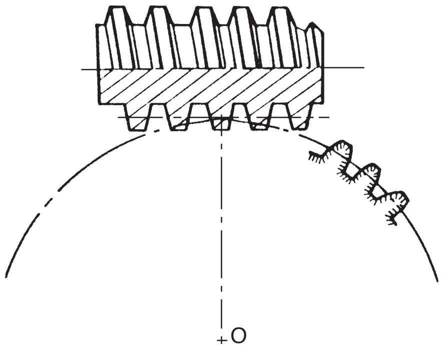


Fig. 9-2 Central Section of a Worm and Worm Gear

To increase the length-of-action, the worm gear is made of a throated shape to wrap around the worm.

9.1.1 Worm Tooth Proportions

Worm tooth dimensions, such as addendum, dedendum, pressure angle, etc., follow the same standards as those for spur and helical gears. The standard values apply to the central section of the mesh. See **Figure 9-3a**. A high pressure angle is favored and in some applications values as high as 25° and 30° are used.

9.1.2 Number Of Threads

The worm can be considered resembling a helical gear with a high helix angle. For extremely high helix angles, there is one continuous tooth or thread. For slightly smaller angles, there can be two, three or even more threads. Thus, a worm is characterized by the number of threads, z_w.

9.1.3 Pitch Diameters, Lead and Lead Angle

Referring to **Figure 9-3**:

$$\text{Pitch diameter of worm} = d_w = \frac{z_w\, p_n}{\pi \sin\gamma} \qquad \textbf{(9-1)}$$

$$\text{Pitch diameter of worm gear} = d_g = \frac{z_g\, p_n}{\pi \cos\gamma} \qquad \textbf{(9-2)}$$

where:

z_w = number of threads of worm; z_g = number of teeth in worm gear

$$L = \text{lead of worm} = z_w\, p_x = \frac{z_w\, p_n}{\cos\gamma}$$

$$\gamma = \text{lead angle} = \tan^{-1}\left(\frac{z_w m}{d_w}\right) = \sin^{-1}\left(\frac{z_w p_n}{\pi\, d_w}\right)$$

$$p_n = p_x \cos\gamma$$

9.1.4 Center Distance

$$C = \frac{d_w + D_g}{2} = \frac{p_n}{2\pi}\left(\frac{z_g}{\cos\gamma} + \frac{z_w}{\sin\gamma}\right) \qquad \textbf{(9-3)}$$

9.2 Cylindrical Worm Gear Calculations

Cylindrical worms may be considered cylindrical type gears with screw threads. Generally, the mesh has a 90° shaft angle. The number of threads in the worm is equivalent to the number of teeth in a gear of a screw type gear mesh. Thus, a one-thread worm is equivalent to a one-tooth gear; and two-threads equivalent to two-teeth, etc. Referring to **Figure 9-4**, for a lead angle γ, measured on the pitch cylinder, each rotation of the worm makes the thread advance one lead.

There are four worm tooth profiles in JIS B 1723, as defined below.

Type I Worm: This worm tooth profile is trapezoid in the radial or axial plane.

Type II Worm: This tooth profile is trapezoid viewed in the normal surface.

Type III Worm: This worm is formed by a cutter in which the tooth profile is trapezoid

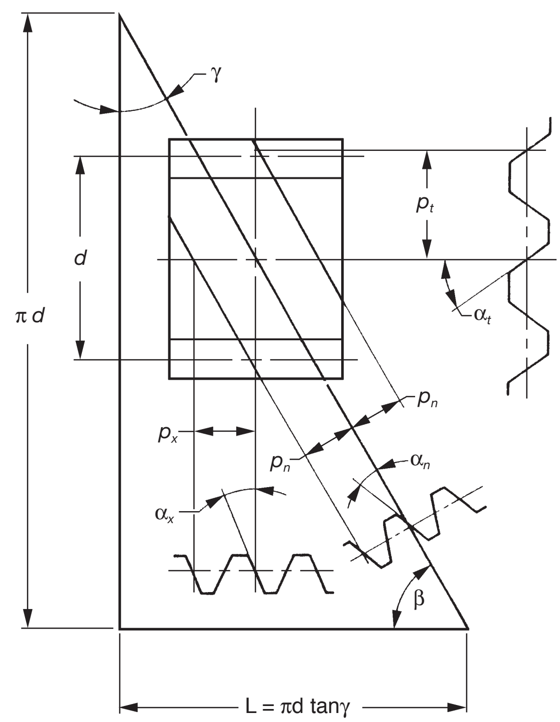
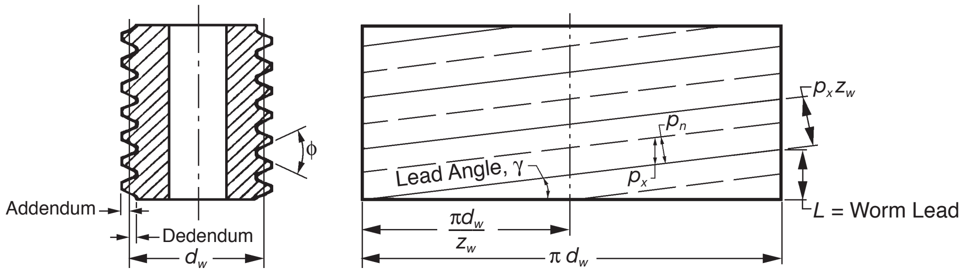


Fig. 9-4 Cylindrical Worm (Right-Hand)



(a) Tooth Proportion of Central Section

(b) Development of Worm's Pitch Cylinder; Two Thread Example: $z_w = 2$

Fig. 9-3 Worm Tooth Proportions and Geometric Relationships

form viewed from the radial surface or axial plane set at the lead angle. Examples are milling and grinding profile cutters.

Type IV Worm: This tooth profile is involute as viewed from the radial surface or at the lead angle. It is an involute helicoid, and is known by that name.

Type III worm is the most popular. In this type, the normal pressure angle α_n has the tendency to become smaller than that of the cutter, α_c.

Per JIS, Type III worm uses a radial module m_t and cutter pressure angle $\alpha_c = 20°$ as the module and pressure angle. A special worm hob is required to cut a Type III worm gear.

Standard values of radial module, m_t, are presented in **Table 9-1**.

Table 9-1 Radial Module of Cylindrical Worm Gears

1	1.25	1.60	2.00	2.50	3.15	4.00	5.00
6.30	8.00	10.00	12.50	16.00	20.00	25.00	—

Because the worm mesh couples nonparallel and nonintersecting axes, the radial surface of the worm, or radial cross section, is the same as the normal surface of the worm gear. Similarly, the normal surface of the worm is the radial surface of the worm gear. The common surface of the worm and worm gear is the normal surface. Using the normal module, m_n, is most popular. Then, an ordinary hob can be used to cut the worm gear.

Table 9-2 presents the relationships among worm and worm gear radial surfaces, normal surfaces, axial surfaces, module, pressure angle, pitch and lead.

Table 9-2 The Relations of Cross Sections of Worm Gears

Worm		
Axial Surface	**Normal Surface**	**Radial Surface**
$m_x = \dfrac{m_n}{\cos\gamma}$	m_n	$m_t = \dfrac{m_n}{\sin\gamma}$
$\alpha_x = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\gamma}\right)$	α_n	$\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$
$p_x = \pi m_x$	$p_n = \pi m_n$	$p_t = \pi m_t$
$L = \pi m_x z_w$	$L = \dfrac{\pi m_n z_w}{\cos\gamma}$	$L = \pi m_t z_w \tan\gamma$
Radial Surface	**Normal Surface**	**Axial Surface**
Worm Gear		

NOTE: The Radial Surface is the plane perpendicular to the axis.

Reference to **Figure 9-4** can help the understanding of the relationships in **Table 9-2**. They are similar to the relations in **Formulas (6-11)** and **(6-12)** that the helix angle β be substituted by $(90° - \gamma)$. We can consider that a worm with lead angle γ is almost the same as a screw gear with helix angle $(90° - \gamma)$.

9.2.1 Axial Module Worm Gears

Table 9-3 presents the equations, for dimensions shown in **Figure 9-5**, for worm gears with axial module, m_x, and normal pressure angle $\alpha_n = 20°$.

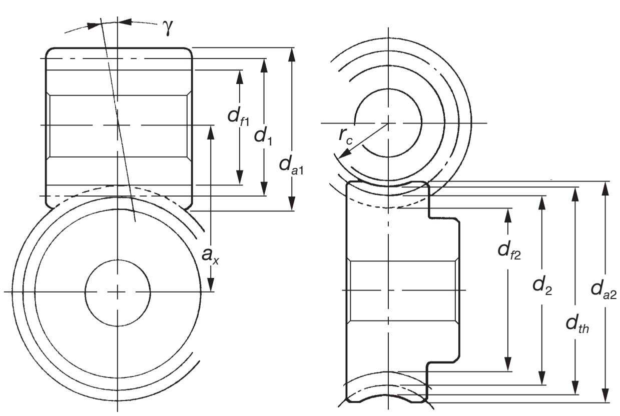


Fig. 9-5 Dimensions of Cylindrical Worm Gears

9.2.2 Normal Module System Worm Gears

The equations for normal module system worm gears are based on a normal module, m_n, and normal pressure angle, $\alpha_n = 20°$. See **Table 9-4**.

9.3 Crowning Of The Worm Gear Tooth

Crowning is critically important to worm gears (worm wheels). Not only can it eliminate abnormal tooth contact due to incorrect assembly, but it also provides for the forming of an oil film, which enhances the lubrication effect of the mesh. This can favorably impact endurance and transmission efficiency of the worm mesh. There are four methods of crowning worm gears:

1. Cut Worm Gear With A Hob Cutter Of Greater Pitch Diameter Than The Worm.

A crownless worm gear results when it is made by using a hob that has an identical pitch diameter as that of the worm. This crownless worm gear is very difficult to assemble correctly. Proper tooth contact and a complete oil film are usually not possible.

However, it is relatively easy to obtain a crowned worm gear by cutting it with a hob whose pitch diameter is slightly larger than that of the worm. This is shown in **Figure 9-6**. This creates teeth contact in the center region with space for oil film formation.

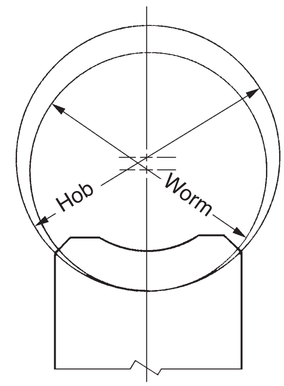


Fig. 9-6 The Method of Using a Greater Diameter Hob

2. Recut With Hob Center Distance Adjustment.

The first step is to cut the worm gear at standard center distance. This results in no crowning. Then the worm gear is finished with the same hob by recutting with the hob axis shifted parallel to the worm gear axis by $\pm\Delta h$. This results in a crowning effect, shown in **Figure 9-7**.

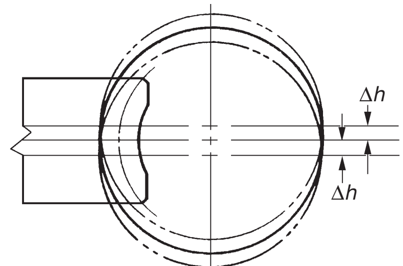


Fig. 9-7 Offsetting Up or Down

3. Hob Axis Inclining $\Delta\theta$ From Standard Position.

In standard cutting, the hob axis is oriented at the proper angle to the worm gear axis. After that, the hob axis is shifted slightly left and then right, $\Delta\theta$, in a plane parallel to the worm gear axis, to cut a crown effect on the worm gear tooth. This is shown in **Figure 9-8**.

Only method **1** is popular. Methods **2** and **3** are seldom used.

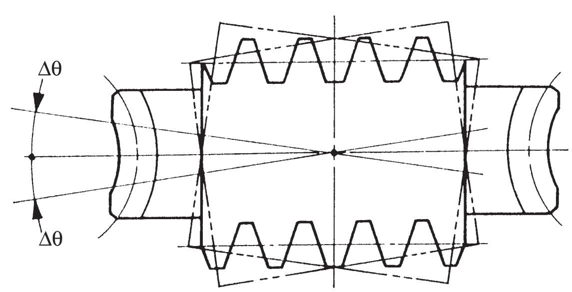


Fig. 9-8 Inclining Right or Left

Table 9-3 The Calculations of Axial Module System Worm Gears (See **Figure 9-5**)

No.	Item	Symbol	Formula	Example Worm	Example Wheel
1	Axial Module	m_x		3	
2	Normal Pressure Angle	α_n		20°	
3	No. of Threads, No. of Teeth	z_w, z_2		∇	30 (R)
4	Standard Pitch Diameter	d_1 d_2	$Q\, m_x$ **Note 1** $z_2 m_x$	44.000	90.000
5	Lead Angle	γ	$\tan^{-1}\left(\frac{m_x z_w}{d_1}\right)$	7.76517°	
6	Coefficient of Profile Shift	x_{a2}		–	0
7	Center Distance	a_x	$\frac{d_1 + d_2}{2} + x_{a2}\, m_x$	67.000	
8	Addendum	h_{a1} h_{a2}	$1.00 m_x$ $(1.00 + x_{a2}) m_x$	3.000	3.000
9	Whole Depth	h	$2.25 m_x$	6.750	
10	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_x$ **Note 2**	50.000	99.000
11	Throat Diameter	d_{th}	$d_2 + 2h_{a2}$	–	96.000
12	Throat Surface Radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
13	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{th} - 2h$	36.500	82.500

∇ Double-Threaded Right-Hand Worm

Note 1: Diameter Factor, Q, means pitch diameter of worm, d_1, over axial module, m_x.

$$Q = \frac{d_1}{m_x}$$

Note 2: There are several calculation methods of worm outside diameter d_{a2} besides those in **Table 9-3**.

Note 3: The length of worm with teeth, b_1, would be sufficient if:
$b_1 = \pi\, m_x\, (4.5 + 0.02 z_2)$

Note 4: Working blank width of worm gear $b_e = 2 m_x \sqrt{(Q + 1)}$. So the actual blank width of $b \geq b_e + 1.5 m_x$ would be enough.

Table 9-4 The Calculations of Normal Module System Worm Gears

No.	Item	Symbol	Formula	Example Worm	Example Worm Gear
1	Normal Module	m_n		3	
2	Normal Pressure Angle	α_n		20°	
3	No. of Threads, No. of Teeth	z_w, z_2		∇	30 (R)
4	Pitch Diameter of Worm	d_1		44.000	–
5	Lead Angle	γ	$\sin^{-1}\left(\frac{m_n z_w}{d_1}\right)$	7.83748°	
6	Pitch Diameter of Worm Gear	d_2	$\frac{z_2 m_n}{\cos\gamma}$	–	90.8486
7	Coefficient of Profile Shift	x_{n2}		–	–0.1414
8	Center Distance	a_x	$\frac{d_1 + d_2}{2} + x_{n2}\, m_n$	67.000	
9	Addendum	h_{a1} h_{a2}	$1.00 m_n$ $(1.00 + x_{n2}) m_n$	3.000	2.5758
10	Whole Depth	h	$2.25 m_n$	6.75	
11	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_n$	50.000	99.000
12	Throat Diameter	d_{th}	$d_2 + 2h_{a2}$	–	96.000
13	Throat Surface Radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
14	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{th} - 2h$	36.500	82.500

∇ Double-Threaded Right-Hand Worm

Note: All notes are the same as those of **Table 9-3**.

4. **Use A Worm With A Larger Pressure Angle Than The Worm Gear.**

This is a very complex method, both theoretically and practically. Usually, the crowning is done to the worm gear, but in this method the modification is on the worm. That is, to change the pressure angle and pitch of the worm without changing the pitch line parallel to the axis, in accordance with the relationships shown in **Equations 9-4**:

$$p_x \cos\alpha_x = p_x' \cos\alpha_x' \qquad \textbf{(9-4)}$$

In order to raise the pressure angle from before change, α_x', to after change, α_x, it is necessary to increase the axial pitch, p_x', to a new value, p_x, per **Equation (9-4)**. The amount of crowning is represented as the space between the worm and worm gear at the meshing point A in **Figure 9-9**. This amount may be approximated by the following equation:

Amount of Crowning

$$= k\,\frac{p_x - p_x'}{p_x'}\,\frac{d_1}{2} \qquad \textbf{(9-5)}$$

where:

d_1 = Pitch diameter of worm
k = Factor from **Table 9-5** and **Figure 9-10**
p_x = Axial pitch after change
p_x' = Axial pitch before change

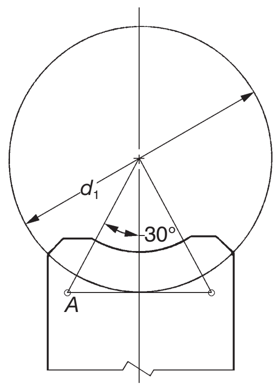


Fig. 9-9 Position A is the Point of Determining Crowning Amount

Table 9-5 The Value of Factor k

α_x	14.5°	17.5°	20°	22.5°
k	0.55	0.46	0.41	0.375

An example of calculating worm crowning is shown in **Table 9-6**.

Because the theory and equations of these methods are so complicated, they are beyond the scope of this treatment. Usually, all stock worm gears are produced with crowning.

Table 9-6 The Calculation of Worm Crowning

No.	Item	Symbol	Formula	Example
	Before Crowning			
1	Axial Module	m_x'		3
2	Normal Pressure Angle	α_n'		20°
3	Number of Threads of Worm	z_w		2
4	Pitch Diameter of Worm	d_1		44.000
5	Lead Angle	γ'	$\tan^{-1}\left(\frac{m_x' z_w}{d_1}\right)$	7.765166°
6	Axial Pressure Angle	α_x'	$\tan^{-1}\left(\frac{\tan\alpha_n'}{\cos\gamma'}\right)$	20.170236°
7	Axial Pitch	p_x'	$\pi m_x'$	9.424778
8	Lead	L'	$\pi m_x' z_v$	18.849556
9	Amount of Crowning	C_R'	*	0.04
10	Factor (k)	k	From **Table 9-5**	0.41
	After Crowning			
11	Axial Pitch	t_x	$t_x'\left(\frac{2C_R}{kd_1} + 1\right)$	9.466573
12	Axial Pressure Angle	α_x	$\cos^{-1}\left(\frac{p_x'}{p_x}\cos\alpha_x'\right)$	20.847973°
13	Axial Module	m_x	$\frac{p_x}{\pi}$	3.013304
14	Lead Angle	γ	$\tan^{-1}\left(\frac{m_x z_w}{d_1}\right)$	7.799179°
15	Normal Pressure Angle	α_n	$\tan^{-1}(\tan\alpha_x \cos\gamma)$	20.671494°
16	Lead	L	$\pi m_x z_w$	18.933146

*It should be determined by considering the size of tooth contact surface.

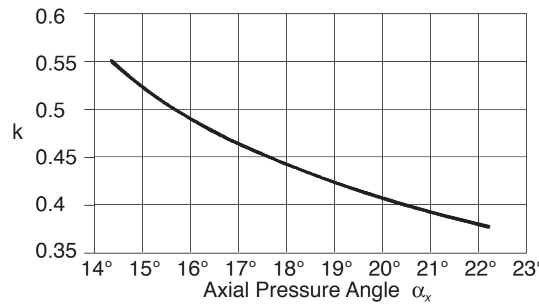


Fig. 9-10 The Value of Factor (*k*)

9.4 Self-Locking Of Worm Mesh

Self-locking is a unique characteristic of worm meshes that can be put to advantage. It is the feature that a worm cannot be driven by the worm gear. It is very useful in the design of some equipment, such as lifting, in that the drive can stop at any position without concern that it can slip in reverse. However, in some situations it can be detrimental if the system requires reverse sensitivity, such as a servomechanism.

Self-locking does not occur in all worm meshes, since it requires special conditions as outlined here. In this analysis, only the driving force acting upon the tooth surfaces is considered without any regard to losses due to bearing friction, lubricant agitation, etc. The governing conditions are as follows:

Let F_{u1} = tangential driving force of worm

$$\text{Then, } F_{u1} = F_n\,(\cos\alpha_n \sin\gamma - \mu\cos\gamma) \qquad \textbf{(9-6)}$$

where:

α_n = normal pressure angle
γ = lead angle of worm
μ = coefficient of friction
F_n = normal driving force of worm

If $F_{u1} > 0$ then there is no self-locking effect at all. Therefore, $F_{u1} \le 0$ is the critical limit of self-locking.

Let α_n in **Equation (9-6)** be 20°, then the condition:

$F_{u1} \le 0$ will become:

$(\cos 20° \sin\gamma - \mu\cos\gamma) \le 0$

Figure 9-11 shows the critical limit of self-locking for lead angle γ and coefficient of friction μ. Practically, it is very hard to assess the exact value of coefficient of friction μ. Further, the bearing loss, lubricant agitation loss, etc. can add many side effects. Therefore, it is not easy to establish precise self-locking conditions. However, it is true that the smaller the lead angle γ, the more likely the self-locking condition will occur.

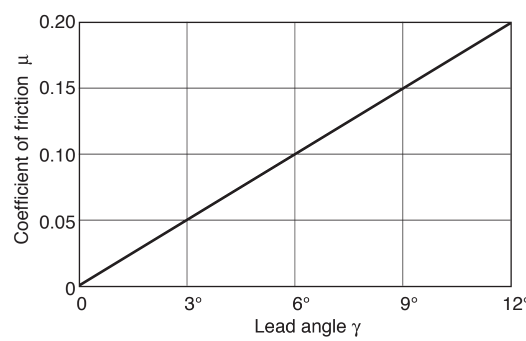


Fig. 9-11 The Critical Limit of Self-locking of Lead Angle γ and Coefficient of Friction μ

SECTION 10 TOOTH THICKNESS

There are direct and indirect methods for measuring tooth thickness. In general, there are three methods:

- Chordal Thickness Measurement
- Span Measurement
- Over Pin or Ball Measurement

10.1 Chordal Thickness Measurement

This method employs a tooth caliper that is referenced from the gear's outside diameter. Thickness is measured at the pitch circle. See **Figure 10-1**.

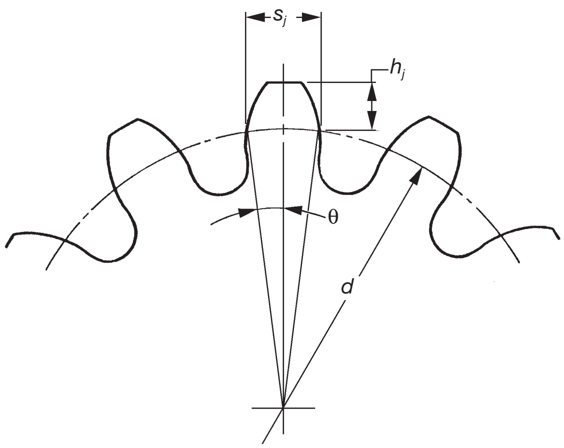


Fig. 10-1 Chordal Thickness Method

10.1.1 Spur Gears

Table 10-1 presents equations for each chordal thickness measurement.

Table 10-1 Equations for Spur Gear Chordal Thickness

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness	s	$\left(\frac{\pi}{2} + 2x\tan\alpha\right)m$	$m = 10$ $\alpha = 20°$ $z = 12$ $x = +0.3$ $h_a = 13.000$ $s = 17.8918$ $\theta = 8.54270°$ $s_j = 17.8256$ $h_j = 13.6657$
2	Half of Tooth Angle at Pitch Circle	θ	$\frac{90}{z} + \frac{360x\tan\alpha}{\pi z}$	
3	Chordal Thickness	s_j	$zm\sin\theta$	
4	Chordal Addendum	h_j	$\frac{zm}{2}(1-\cos\theta) + h_a$	

10.1.2 Spur Racks And Helical Racks

The governing equations become simple since the rack tooth profile is trapezoid, as shown in **Table 10-2**.

Table 10-2 Chordal Thickness of Racks

No.	Item	Symbol	Formula	Example
1	Chordal Thickness	s_j	$\frac{\pi m}{2}$ or $\frac{\pi m_n}{2}$	$m = 3$ $\alpha = 20°$ $s_j = 4.7124$ $h_a = 3.0000$
2	Chordal Addendum	h_j	h_a	

NOTE: These equations are also applicable to helical racks.

10.1.3 Helical Gears

The chordal thickness of helical gears should be measured on the normal surface basis as shown in **Table 10-3**. **Table 10-4** presents the equations for chordal thickness of helical gears in the radial system.

10.1.4 Bevel Gears

Table 10-5 shows the equations of chordal thickness for a Gleason straight bevel gear.

Table 10-6 presents equations for chordal thickness of a standard straight bevel gear.

If a standard straight bevel gear is cut by a Gleason straight bevel cutter, the tooth angle should be adjusted according to:

$$\text{tooth angle } (°) = \frac{180°}{\pi R_e}\left(\frac{s}{2} + h_f\tan\alpha\right) \qquad \textbf{(10-1)}$$

This angle is used as a reference in determining the circular tooth thickness, s, in setting up the gear cutting machine.

Table 10-7 presents equations for chordal thickness of a Gleason spiral bevel gear.

The calculations of circular thickness of a Gleason spiral bevel gear are so complicated that we do not intend to go further in this presentation.

10.1.5 Worms And Worm Gears

Table 10-8 presents equations for chordal thickness of axial module worms and worm gears.

Table 10-9 contains the equations for chordal thickness of normal module worms and worm gears.

10.2 Span Measurement Of Teeth

Span measurement of teeth, s_m, is a measure over a number of teeth, z_m, made by means of a special tooth thickness micrometer. The value measured is the sum of normal circular tooth thickness on the base circle, s_{bn}, and normal pitch, $p_{en}(z_m - 1)$.

10.2.1 Spur And Internal Gears

The applicable equations are presented in **Table 10-10**.

Figure 10-4 shows the span measurement of a spur gear. This measurement is on the outside of the teeth.

For internal gears the tooth profile is opposite to that of the external spur gear. Therefore, the measurement is between the inside of the tooth profiles.

10.2.2 Helical Gears

Tables 10-11 and **10-12** present equations for span measurement of the normal and the radial systems, respectively, of helical gears.

Table 10-3 Equations for Chordal Thickness of Helical Gears in the Normal System

No.	Item	Symbol	Formula	Example
1	Normal Circular Tooth Thickness	s_n	$\left(\frac{\pi}{2} + 2x_n \tan\alpha_n\right) m_n$	$m_n = 5$ $\alpha_n = 20°$ $\beta = 25°\ 00'\ 00''$ $z = 16$ $x_n = +0.2$ $h_a = 6.0000$ $s_n = 8.5819$ $z_v = 21.4928$ $\theta_v = 4.57556°$ $s_j = 8.5728$ $h_j = 6.1712$
2	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Half of Tooth Angle at Pitch Circle	θ_v	$\frac{90}{z_v} + \frac{360 x_n \tan\alpha_n}{\pi z_v}$	
4	Chordal Thickness	s_j	$z_v m_n \sin\theta_v$	
5	Chordal Addendum	h_j	$\frac{z_v m_n}{2}(1 - \cos\theta_v) + h_a$	

Table 10-4 Equations for Chordal Thickness of Helical Gears in the Radial System

No.	Item	Symbol	Formula	Example
1	Normal Circular Tooth Thickness	s_n	$\left(\frac{\pi}{2} + 2x_t \tan\alpha_t\right) m_t \cos\beta$	$m = 4$ $\alpha_t = 20°$ $\beta = 22°\ 30'\ 00''$ $z = 20$ $x_t = +0.3$ $h_a = 4.7184$ $s_n = 6.6119$ $z_v = 25.3620$ $\theta_v = 4.04196°$ $s_j = 6.6065$ $h_j = 4.8350$
2	Number of Teeth in an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Half of Tooth Angle at Pitch Circle	θ_v	$\frac{90}{z_v} + \frac{360 x_t \tan\alpha_t}{\pi z_v}$	
4	Chordal Thickness	s_j	$z_v m_t \cos\beta \sin\theta_v$	
5	Chordal Addendum	h_j	$\frac{z_v m_t \cos\beta}{2}(1 - \cos\theta_v) + h_a$	

NOTE: Table 10-4 equations are also for the tooth profile of a Sunderland gear.

Table 10-5 Equations for Chordal Thickness of Gleason Straight Bevel Gears

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness Factor (Coefficient of Horizontal Profile Shift)	K	Obtain from **Figure 10-2** below	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $\frac{z_1}{z_2} = 0.4$ $K = 0.0259$ $h_{a1} = 5.5456$ $h_{a2} = 2.4544$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s_1 = 7.5119$ $s_2 = 5.0545$ $s_{j1} = 7.4946$ $s_{j2} = 5.0536$ $h_{j1} = 5.7502$ $h_{j2} = 2.4692$
2	Circular Tooth Thickness	s_1 s_2	$\pi m - s_2$ $\frac{\pi m}{2} - (h_{a1} - h_{a2})\tan\alpha - Km$	
4	Chordal Thickness	s_j	$s - \frac{s^3}{6d^2}$	
5	Chordal Addendum	h_j	$h_a + \frac{s^2 \cos\delta}{4d}$	

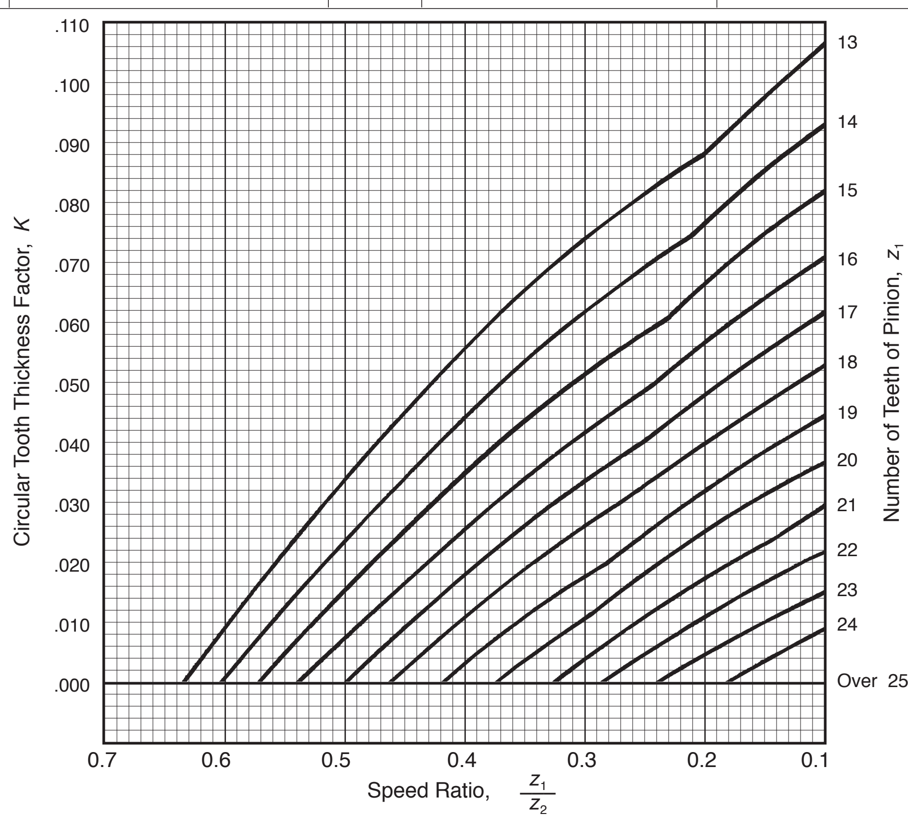


Fig. 10-2 Chart to Determine the Circular Tooth Thickness Factor K for Gleason Straight Bevel Gear (See **Table 10-5**)

Table 10-6 Equations for Chordal Thickness of Standard Straight Bevel Gears

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness	s	$\frac{\pi m}{2}$	$m = 4$ $\alpha = 20°$ $\quad \Sigma = 90°$ $z_1 = 16$ $\quad z_2 = 40$ $d_1 = 64$ $\quad d_2 = 160$ $h_a = 4.0000$ $\delta_1 = 21.8014°$ $\quad \delta_2 = 68.1986°$ $s = 6.2832$ $z_{v1} = 17.2325$ $\quad z_{v2} = 107.7033$ $R_{v1} = 34.4650$ $\quad R_{v2} = 215.4066$ $\theta_{v1} = 5.2227°$ $\quad \theta_{v2} = 0.83563°$ $s_{j1} = 6.2745$ $\quad s_{j2} = 6.2830$ $h_{j1} = 4.1431$ $\quad h_{j2} = 4.0229$
2	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos\delta}$	
3	Back Cone Distance	R_v	$\frac{d}{2\cos\delta}$	
4	Half of Tooth Angle at Pitch Circle	θ_v	$\frac{90}{z_v}$	
5	Chordal Thickness	s_j	$z_v m \sin\theta_v$	
6	Chordal Addendum	h_j	$h_a + R_v(1 - \cos\theta_v)$	

Table 10-7 Equations for Chordal Thickness of Gleason Spiral Bevel Gears

No.	Item	Symbol	Formula	Example
1	Circular Tooth Thickness Factor	K	Obtain from **Figure 10-3**	$\Sigma = 90°$ $\quad m = 3$ $\quad \alpha_n = 20°$ $z_1 = 20$ $\quad z_2 = 40$ $\quad \beta_m = 35°$ $h_{a1} = 3.4275$ $\quad h_{a2} = 1.6725$ $K = 0.060$ $p = 9.4248$ $s_1 = 5.6722$ $\quad s_2 = 3.7526$
2	Circular Tooth Thickness	s_1 s_2	$p - s_2$ $\frac{p}{2} - (h_{a1} - h_{a2})\frac{\tan\alpha_n}{\cos\beta_m} - Km$	

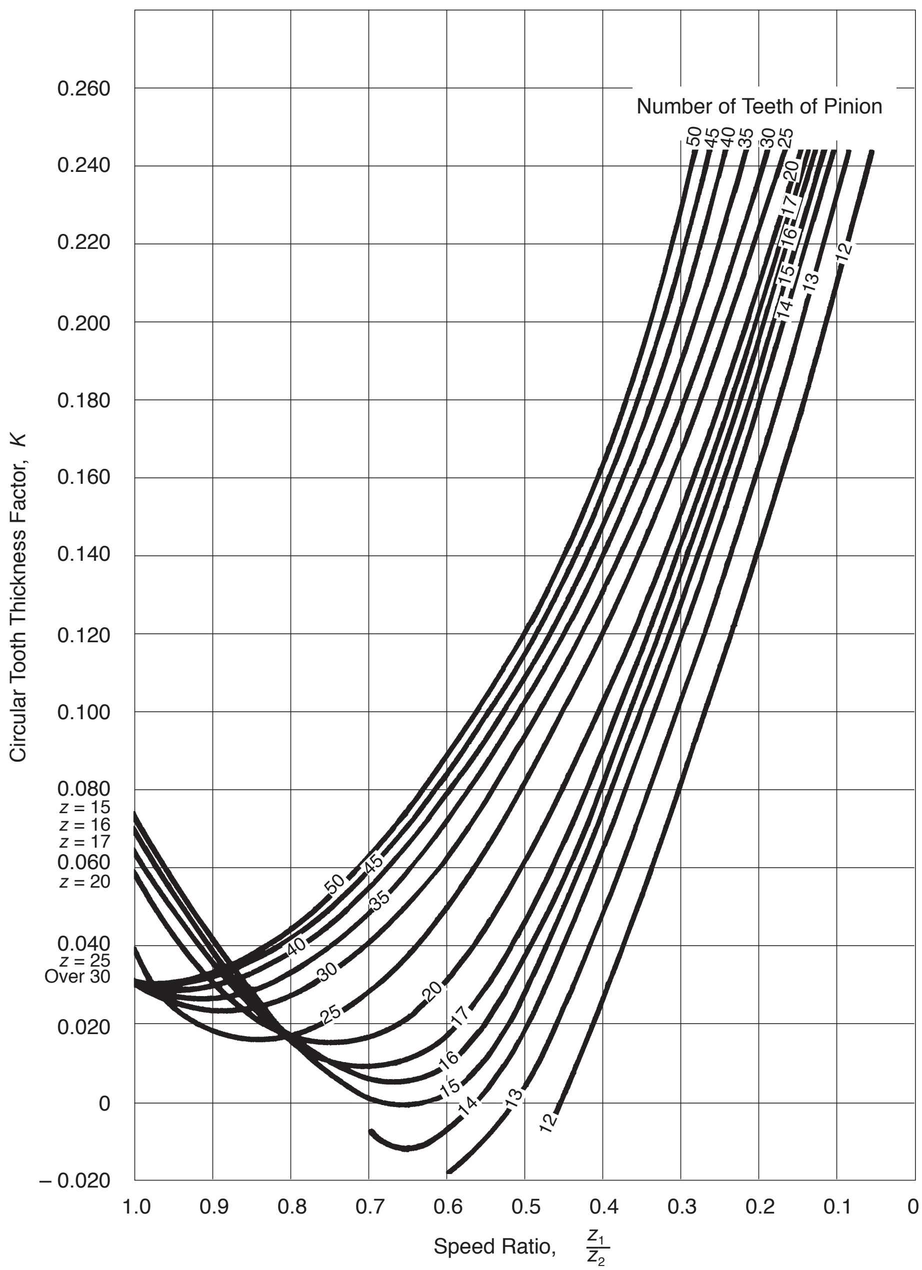


Fig. 10-3 Chart to Determine the Circular Tooth Thickness Factor *K* for Gleason Spiral Bevel Gears

Table 10-8 Equations for Chordal Thickness of Axial Module Worms and Worm Gear

No.	Item	Symbol	Formula	Example
1	Axial Circular Tooth Thickness of Worm Radial Circular Tooth Thickness of Worm Gear	s_{x1} s_{x2}	$\dfrac{\pi m_x}{2}$ $(\dfrac{\pi}{2} + 2x_{x2} \tan\alpha_x) m_x$	$m_x = 3$ $\alpha_n = 20°$ $z_w = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 90$ $a_x = 65$ $x_{x2} = +0.33333$ $h_{a1} = 3.0000$ $h_{a2} = 4.0000$ $\gamma = 8.97263°$ $\alpha_x = 20.22780°$ $s_{x1} = 4.71239$ $s_{x2} = 5.44934$ $z_{v2} = 31.12885$ $\theta_{v2} = 3.34335°$ $s_{j1} = 4.6547$ $s_{j2} = 5.3796$ $h_{j1} = 3.0035$ $h_{j2} = 4.0785$
2	No. of Teeth in an Equivalent Spur Gear (Worm Gear)	z_{v2}	$\dfrac{z_2}{\cos^3\gamma}$	
3	Half of Tooth Angle at Pitch Circle (Worm Gear)	θ_{v2}	$\dfrac{90}{z_{v2}} + \dfrac{360\, x_{x2} \tan\alpha_x}{\pi z_{v2}}$	
4	Chordal Thickness	s_{j1} s_{j2}	$s_{x1} \cos\gamma$ $z_v m_x \cos\gamma \sin\theta_{v2}$	
5	Chordal Addendum	h_{j1} h_{j2}	$h_{a1} + \dfrac{(s_{x1} \sin\gamma \cos\gamma)^2}{4d_1}$ $h_{a2} + \dfrac{z_v m_x \cos\gamma}{2}(1 - \cos\theta_{v2})$	

Table 10-9 Equations for Chordal Thickness of Normal Module Worms and Worm Gears

No.	Item	Symbol	Formula	Example
1	Axial Circular Tooth Thickness of Worm Radial Circular Tooth Thickness of Worm Gear	s_{n1} s_{n2}	$\dfrac{\pi m_n}{2}$ $(\dfrac{\pi}{2} + 2x_{n2} \tan\alpha_n) m_n$	$m_n = 3$ $\alpha_n = 20°$ $z_w = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 91.1433$ $a_x = 65$ $x_{n2} = 0.14278$ $h_{a1} = 3.0000$ $h_{a2} = 3.42835$ $\gamma = 9.08472°$ $s_{n1} = 4.71239$ $s_{n2} = 5.02419$ $z_{v2} = 31.15789$ $\theta_{v2} = 3.07964°$ $s_{j1} = 4.7124$ $s_{j2} = 5.0218$ $h_{j1} = 3.0036$ $h_{j2} = 3.4958$
2	No. of Teeth in an Equivalent Spur Gear (Worm Gear)	z_{v2}	$\dfrac{z_2}{\cos^3\gamma}$	
3	Half of Tooth Angle at Pitch Circle (Worm Gear)	θ_{v2}	$\dfrac{90}{z_{v2}} + \dfrac{360\, x_{n2} \tan\alpha_n}{\pi z_{v2}}$	
4	Chordal Thickness	s_{j1} s_{j2}	$s_{n1} \cos\gamma$ $z_v m_n \cos\gamma \sin\theta_{v2}$	
5	Chordal Addendum	h_{j1} h_{j2}	$h_{a1} + \dfrac{(s_{n1} \sin\gamma)^2}{4d_1}$ $h_{a2} + \dfrac{z_v m_n \cos\gamma}{2}(1 - \cos\theta_{v2})$	

Table 10-10 Span Measurement of Spur and Internal Gear Teeth

No.	Item	Symbol	Formula	Example
1	Span Number of Teeth	z_m	$z_{mth} = zK(f) + 0.5$ See **NOTE** Select the nearest natural number of z_{mth} as z_m.	$m = 3$ $\alpha = 20°$ $z = 24$ $x = +0.4$ $z_{mth} = 3.78787$ $z_m = 4$ $s_m = 32.8266$
2	Span Measurement	s_m	$m \cos\alpha\ [\pi (z_m - 0.5) + z \operatorname{inv}\alpha] + 2xm \sin\alpha$	

NOTE:

$$K(f) = \frac{1}{\pi}\left[\sec\alpha \sqrt{(1 + 2f)^2 - \cos^2\alpha} - \operatorname{inv}\alpha - 2f \tan\alpha\right] \qquad \textbf{(10-2)}$$

where $f = \dfrac{x}{z}$

Table 10-11 Equations for Span Measurement of the Normal System Helical Gears

No.	Item	Symbol	Formula	Example
1	Span Number of Teeth	z_m	$z_{mth} = zK(f, \beta) + 0.5$ See **NOTE** Select the nearest natural number of z_{mth} as z_m.	$m_n = 3$, $\alpha_n = 20°$, $z = 24$ $\beta = 25°\ 00'\ 00''$ $x_n = +0.4$ $\alpha_s = 21.88023°$ $z_{mth} = 4.63009$ $z_m = 5$ $s_m = 42.0085$
2	Span Measurement	s_m	$m_n \cos\alpha_n\ [\pi (z_m - 0.5) + z \operatorname{inv}\alpha_t]$ $+ 2x_n m_n \sin\alpha_n$	

NOTE:

$$K(f, \beta) = \frac{1}{\pi}\left[\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right)\sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \operatorname{inv}\alpha_t - 2f \tan\alpha_n\right] \qquad \textbf{(10-3)}$$

where $f = \dfrac{x_n}{z}$

Table 10-12 Equations for Span Measurement of the Radial System Helical Gears

No.	Item	Symbol	Formula	Example
1	Span Number of Teeth	z_m	$z_{mth} = zK(f,\beta) + 0.5$ See **NOTE** Select the nearest natural number of z_{mth} as z_m.	$m_t = 3$, $\alpha_t = 20°$, $z = 24$ $\beta = 22°\ 30'\ 00''$ $x_t = +0.4$ $\alpha_n = 18.58597°$
2	Span Measurement	s_m	$m_t \cos\beta \cos\alpha_n\ [\pi\ (z_m - 0.5) + z\ \mathrm{inv}\alpha_t\ ] + 2x_t m_t \sin\alpha_n$	$z_{mth} = 4.31728$ $z_m = 4$ $s_m = 30.5910$

NOTE:

$$K(f,\beta) = \frac{1}{\pi}\ \left[\left(1 + \frac{\sin^2\beta}{\cos^2\beta + \tan^2\alpha_n}\right) \sqrt{(\cos^2\beta + \tan^2\alpha_n)(\sec\beta + 2f)^2 - 1} - \mathrm{inv}\alpha_t - 2f\tan\alpha_n\right] \quad (\mathbf{10\text{-}4})$$

where $f = \dfrac{x_t}{z\cos\beta}$

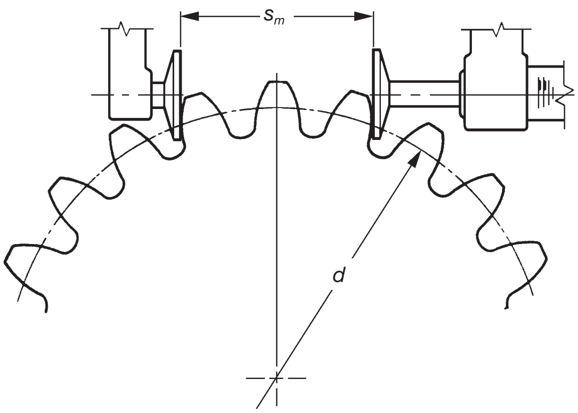


Fig. 10-4 Span Measurement of Teeth (Spur Gear)

There is a requirement of a minimum blank width to make a helical gear span measurement. Let b_{min} be the minimum value for blank width. See **Figure 10-5**. Then

$$b_{min} = s_m \sin\beta_b + \Delta b \quad (\mathbf{10\text{-}5})$$

where β_b is the helix angle at the base cylinder,

$$\begin{aligned}\beta_b &= \tan^{-1}(\tan\beta\cos\alpha_t)\\ &= \sin^{-1}(\sin\beta\cos\alpha_n)\end{aligned} \quad (\mathbf{10\text{-}6})$$

From the above, we can determine that $\Delta b > 3$ mm to make a stable measurement of s_m.

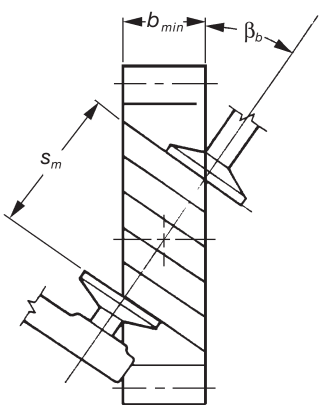


Fig. 10-5 Blank Width of Helical Gear

10.3 Over Pin (Ball) Measurement

As shown in **Figures 10-6** and **10-7**, measurement is made over the outside of two pins that are inserted in diametrically opposite tooth spaces, for even tooth number gears; and as close as possible for odd tooth number gears.

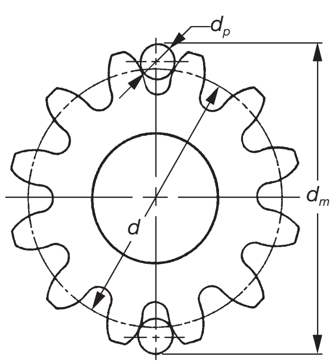


Fig. 10-6 Even Number of Teeth

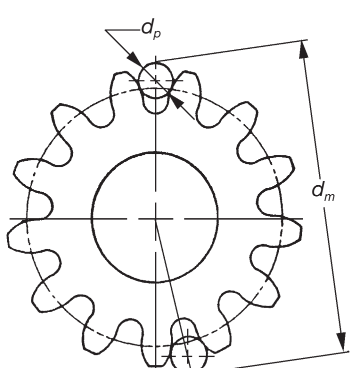


Fig. 10-7 Odd Number of Teeth

The procedure for measuring a rack with a pin or a ball is as shown in **Figure 10-9** by putting pin or ball in the tooth space and using a micrometer between it and a reference surface. Internal gears are similarly measured, except that the measurement is between the pins. See **Figure 10-10**. Helical gears can only be measured with balls. In the case of a worm, three pins are used, as shown in **Figure 10-11**. This is similar to the procedure of measuring a screw thread. All these cases are discussed in detail in the following sections.

Note that gear literature uses “over pins” and “over wires” terminology interchangeably. The “over wires” term is often associated with very fine pitch gears because the diameters are accordingly small.

10.3.1 Spur Gears

In measuring a gear, the size of the pin must be such that the over pins measurement is larger than the gear's outside diameter. An ideal value is one that would place the point of contact (tangent point) of pin and tooth profile at the pitch radius. However, this is not a necessary requirement. Referring to **Figure 10-8**, following are the equations for calculating the over pins measurement for a specific tooth thickness, s, regardless of where the pin contacts the tooth profile:

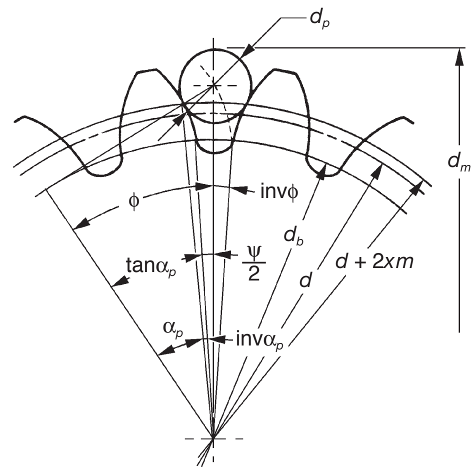


Fig. 10-8 Over Pins Diameter of Spur Gear

For even number of teeth:

$$d_m = \frac{d\cos\phi}{\cos\phi_1} + d_p \quad (\mathbf{10\text{-}7})$$

For odd number of teeth:

$$d_m = \frac{d\cos\phi}{\cos\phi_1}\cos\left(\frac{90°}{z}\right) + d_p \quad (\mathbf{10\text{-}8})$$

where the value of ϕ_1 is obtained from:

$$\mathrm{inv}\phi_1 = \frac{s}{d} + \mathrm{inv}\phi + \frac{d_p}{d\cos\phi} - \frac{\pi}{z} \quad (\mathbf{10\text{-}9})$$

When tooth thickness, s, is to be calculated from a known over pins measurement, d_m, the above equations can be manipulated to yield:

$$s = d\left(\frac{\pi}{z} + \mathrm{inv}\phi_c - \mathrm{inv}\phi + \frac{d_p}{d\cos\phi}\right) \quad (\mathbf{10\text{-}10})$$

where

$$\cos\phi_c = \frac{d\cos\phi}{2R_c} \quad (\mathbf{10\text{-}11})$$

For even number of teeth:

$$R_c = \frac{d_m - d_p}{2} \quad (\mathbf{10\text{-}12})$$

For odd number of teeth:

$$R_c = \frac{d_m - d_p}{2\cos(\frac{90°}{z})} \qquad \textbf{(10-13)}$$

In measuring a standard gear, the size of the pin must meet the condition that its surface should have the tangent point at the standard pitch circle. While, in measuring a shifted gear, the surface of the pin should have the tangent point at the $d + 2xm$ circle. The ideal diameters of pins when calculated from the equations of **Table 10-13** may not be practical. So, in practice, we select a standard pin diameter close to the ideal value. After the actual diameter of pin d_p is determined, the over pin measurement d_m can be calculated from **Table 10-14**.

Table 10-15 is a dimensional table under the condition of module $m = 1$ and pressure angle $\alpha = 20°$ with which the pin has the tangent point at $d + 2xm$ circle.

10.3.2 Spur Racks And Helical Racks

In measuring a rack, the pin is ideally tangent with the tooth flank at the pitch line. The equations in **Table 10-16** can, thus, be derived. In the case of a helical rack, module m, and pressure angle α, in **Table 10-16**, can be substituted by normal module, m_n, and normal pressure angle, α_n, resulting in **Table 10-16A**.

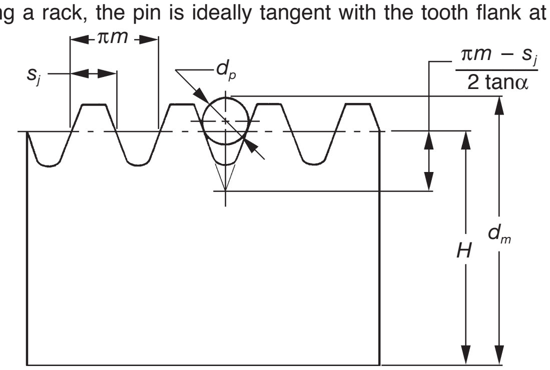


Fig. 10-9 Over Pins Measurement for a Rack Using a Pin or a Ball

10.3.3 Internal Gears

As shown in **Figure 10-10**, measuring an internal gear needs a proper pin which has its tangent point at $d + 2xm$ circle. The equations are in **Table 10-17** for obtaining the ideal pin diameter. The equations for calculating the between pin measurement, d_m, are given in **Table 10-18**.

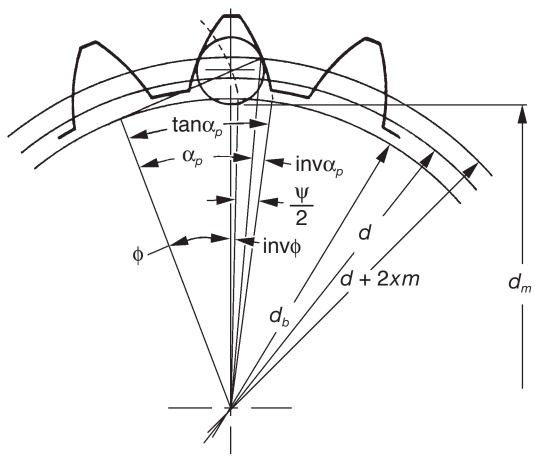


Fig. 10-10 Between Pin Dimension of Internal Gears

Table 10-19 lists ideal pin diameters for standard and profile shifted gears under the condition of module $m = 1$ and pressure angle $\alpha = 20°$, which makes the pin tangent to the pitch circle $d + 2xm$.

10.3.4 Helical Gears

The ideal pin that makes contact at the $d + 2x_n m_n$ pitch circle of a helical gear can be obtained from the same above equations, but with the teeth number z substituted by the equivalent (virtual) teeth number z_v.

Table 10-20 presents equations for deriving over pin diameters.

Table 10-21 presents equations for calculating over pin measurements for helical gears in the normal system.

Table 10-22 and **Table 10-23** present equations for calculating pin measurements for helical gears in the radial (perpendicular to axis) system.

10.3.5 Three Wire Method Of Worm Measurement

The teeth profile of Type III worms which are most popular are cut by standard cutters with a pressure angle $\alpha_c = 20°$. This results in the normal pressure angle of the worm being a bit smaller than 20°. The equation below shows how to calculate a Type III worm in an AGMA system.

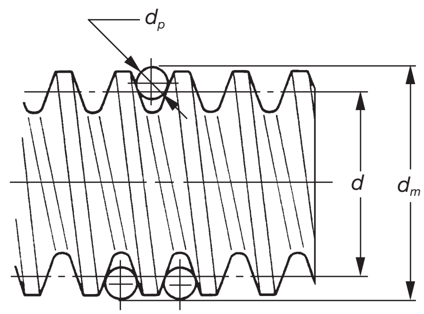


Fig. 10-11 Three Wire Method of a Worm

$$\alpha_n = \alpha_c - \frac{90°}{z_w}\,\frac{r}{r_c\cos^2\gamma + r}\,\sin^3\gamma \qquad \textbf{(10-14)}$$

where:

r = Worm Pitch Radius
r_c = Cutter Radius
z_w = Number of Threads
γ = Lead Angle of Worm

The exact equation for a three wire method of Type III worm is not only difficult to comprehend, but also hard to calculate precisely. We will introduce two approximate calculation methods here:

(a) Regard the tooth profile of the worm as a linear tooth profile of a rack and apply its equations. Using this system, the three wire method of a worm can be calculated by **Table 10-24**.

These equations presume the worm lead angle to be very small and can be neglected. Of course, as the lead angle gets larger, the equations' error gets correspondingly larger. If the lead angle is considered as a factor, the equations are as in **Table 10-25**.

(b) Consider a worm to be a helical gear.

This means applying the equations for calculating over pins measurement of helical gears to the case of three wire method of a worm. Because the tooth profile of Type III worm is not an involute curve, the method yields an approximation. However, the accuracy is adequate in practice.

Tables 10-26 and **10-27** contain equations based on the axial system. **Tables 10-28** and **10-29** are based on the normal system.

Table 10-28 shows the calculation of a worm in the normal module system. Basically, the normal module system and the axial module system have the same form of equations. Only the notations of module make them different.

10.4 Over Pins Measurements For Fine Pitch Gears With Specific Numbers Of Teeth

Table 10-30 presents measurements for metric gears. These are for standard ideal tooth thicknesses. Measurements can be adjusted accordingly to backlash allowance and tolerance; i.e., tooth thinning.

Table 10-13 Equations for Calculating Ideal Pin Diameters

No.	Item	Symbol	Formula	Example
1	Half Tooth Space Angle at Base Circle	$\frac{\psi}{2}$	$\left(\frac{\pi}{2z} - \text{inv}\alpha\right) - \frac{2x\tan\alpha}{z}$	m = 1 α = 20° z = 20 x = 0 $\frac{\psi}{2}$ = 0.0636354 α_p = 20° ϕ = 0.4276057 d_p = 1.7245
2	The Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_p	$\cos^{-1}\left[\frac{zm\cos\alpha}{(z+2x)m}\right]$	
3	The Pressure Angle at Pin Center	ϕ	$\tan\alpha_p + \frac{\psi}{2}$	
4	Ideal Pin Diameter	d_p	$zm\cos\alpha\,(\text{inv}\phi + \frac{\psi}{2})$	

NOTE: The units of angles $\psi/2$ and ϕ are radians.

Table 10-14 Equations for Over Pins Measurement for Spur Gears

No.	Item	Symbol	Formula	Example
1	Actual Diameter of Pin	d_p	See **NOTE**	Let d_p = 1.7, then: invϕ = 0.0268197 ϕ = 24.1350° d_m = 22.2941
2	Involute Function ϕ	invϕ	$\frac{d_p}{mz\cos\alpha} - \frac{\pi}{2z} + \text{inv}\alpha + \frac{2x\tan\alpha}{z}$	
3	The Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Over Pins Measurement	d_m	Even Teeth $\frac{zm\cos\alpha}{\cos\phi} + d_p$	
			Odd Teeth $\frac{zm\cos\alpha}{\cos\phi}\cos\frac{90°}{z} + d_p$	

NOTE: The value of the ideal pin diameter from **Table 10-13**, or its approximate value, is applied as the actual diameter of pin d_p here.

Table 10-15 The Size of Pin which Has the Tangent Point at d + 2xm Circle of Spur Gears

Number of Teeth z	Coefficient of Profile Shift, x m = 1, α = 20°							
	– 0.4	– 0.2	0	0.2	0.4	0.6	0.8	1.0
10	—	1.6348	1.7886	1.9979	2.2687	2.6079	3.0248	3.5315
20	1.6231	1.6599	1.7245	1.8149	1.9306	2.0718	2.2389	2.4329
30	1.6418	1.6649	1.7057	1.7632	1.8369	1.9267	2.0324	2.1542
40	1.6500	1.6669	1.6967	1.7389	1.7930	1.8589	1.9365	2.0257
50	1.6547	1.6680	1.6915	1.7248	1.7675	1.8196	1.8810	1.9516
60	1.6577	1.6687	1.6881	1.7155	1.7509	1.7940	1.8448	1.9032
70	1.6598	1.6692	1.6857	1.7090	1.7392	1.7759	1.8193	1.8691
80	1.6614	1.6695	1.6839	1.7042	1.7305	1.7625	1.8003	1.8438
90	1.6625	1.6698	1.6825	1.7005	1.7237	1.7521	1.7857	1.8242
100	1.6635	1.6700	1.6814	1.6975	1.7184	1.7439	1.7740	1.8087
110	1.6642	1.6701	1.6805	1.6951	1.7140	1.7372	1.7645	1.7960
120	1.6649	1.6703	1.6797	1.6931	1.7104	1.7316	1.7567	1.7855
130	1.6654	1.6704	1.6791	1.6914	1.7074	1.7269	1.7500	1.7766
140	1.6659	1.6705	1.6785	1.6900	1.7048	1.7229	1.7444	1.7690
150	1.6663	1.6706	1.6781	1.6887	1.7025	1.7195	1.7394	1.7625
160	1.6666	1.6706	1.6777	1.6877	1.7006	1.7164	1.7351	1.7567
170	1.6669	1.6707	1.6773	1.6867	1.6989	1.7138	1.7314	1.7517
180	1.6672	1.6708	1.6770	1.6858	1.6973	1.7114	1.7280	1.7472
190	1.6674	1.6708	1.6767	1.6851	1.6960	1.7093	1.7250	1.7432
200	1.6676	1.6708	1.6764	1.6844	1.6947	1.7074	1.7223	1.7396

Table 10-16 Equations for Over Pins Measurement of Spur Racks

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\frac{\pi m - s_j}{\cos\alpha}$	m = 1 α = 20° s_j = 1.5708 Ideal Pin Diameter d_p' = 1.6716 Actual Pin Diameter d_p = 1.7 H = 14.0000 d_m = 15.1774
2	Over Pins Measurement	d_m	$H - \frac{\pi m - s_j}{2\tan\alpha} + \frac{d_p}{2}\left(1 + \frac{1}{\sin\alpha}\right)$	

Table 10-16A Equations for Over Pins Measurement of Helical Racks

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\frac{\pi m_n - s_j}{\cos\alpha_n}$	m_n = 1 α_n = 20° s_j = 1.5708 β = 15° Ideal Pin Diameter d_p' = 1.6716 Actual Pin Diameter d_p = 1.7 H = 14.0000 d_m = 15.1774
2	Over Pins Measurement	d_m	$H - \frac{\pi m_n - s_j}{2\tan\alpha_n} + \frac{d_p}{2}\left(1 + \frac{1}{\sin\alpha_n}\right)$	

Table 10-17 Equations for Calculating Pin Size for Internal Gears

No.	Item	Symbol	Formula	Example
1	Half of Tooth Space Angle at Base Circle	$\frac{\psi}{2}$	$\left(\frac{\pi}{2z} + \text{inv}\alpha\right) + \frac{2x\tan\alpha}{z}$	$m = 1$ $\alpha = 20°$ $z = 40$ $x = 0$ $\frac{\psi}{2} = 0.054174$ $\alpha_p = 20°$ $\phi = 0.309796$ $d_p = 1.6489$
2	The Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_p	$\cos^{-1}\left[\frac{zm\cos\alpha}{(z+2x)m}\right]$	
3	The Pressure Angle at Pin Center	ϕ	$\tan\alpha_p - \frac{\psi}{2}$	
4	Ideal Pin Diameter	d_p	$zm\cos\alpha\left(\frac{\psi}{2} - \text{inv}\phi\right)$	

NOTE: The units of angles $\psi/2$ and ϕ are radians.

Table 10-18 Equations for Between Pins Measurement of Internal Gears

No.	Item	Symbol	Formula	Example
1	Actual Diameter of Pin	d_p	See **NOTE**	
2	Involute Function ϕ	$\text{inv}\phi$	$\left(\frac{\pi}{2z} + \text{inv}\alpha\right) - \frac{d_p}{zm\cos\alpha} + \frac{2x\tan\alpha}{z}$	Let $d_p = 1.7$, then:
3	The Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	$\text{inv}\phi = 0.0089467$ $\phi = 16.9521°$ $d_m = 37.5951$
4	Between Pins Measurement	d_m	Even Teeth $\frac{zm\cos\alpha}{\cos\phi} - d_p$ Odd Teeth $\frac{zm\cos\alpha}{\cos\phi}\cos\frac{90°}{z} - d_p$	

NOTE: First, calculate the ideal pin diameter. Then, choose the nearest practical actual pin size.

Table 10-19 The Size of Pin that is Tangent at Pitch Circle d + 2xm of Internal Gears

Number of Teeth z	Coefficient of Profile Shift, x m = 1, $\alpha = 20°$							
	– 0.4	– 0.2	0	0.2	0.4	0.6	0.8	1.0
10	—	1.4789	1.5936	1.6758	1.7283	1.7519	1.7460	1.7092
20	1.4687	1.5604	1.6284	1.6759	1.7047	1.7154	1.7084	1.6837
30	1.5309	1.5942	1.6418	1.6751	1.6949	1.7016	1.6956	1.6771
40	1.5640	1.6123	1.6489	1.6745	1.6895	1.6944	1.6893	1.6744
50	1.5845	1.6236	1.6533	1.6740	1.6862	1.6900	1.6856	1.6732
60	1.5985	1.6312	1.6562	1.6737	1.6839	1.6870	1.6832	1.6725
70	1.6086	1.6368	1.6583	1.6734	1.6822	1.6849	1.6815	1.6721
80	1.6162	1.6410	1.6600	1.6732	1.6810	1.6833	1.6802	1.6718
90	1.6222	1.6443	1.6612	1.6731	1.6800	1.6820	1.6792	1.6717
100	1.6270	1.6470	1.6622	1.6729	1.6792	1.6810	1.6784	1.6716
110	1.6310	1.6492	1.6631	1.6728	1.6785	1.6801	1.6778	1.6715
120	1.6343	1.6510	1.6638	1.6727	1.6779	1.6794	1.6772	1.6714
130	1.6371	1.6525	1.6644	1.6727	1.6775	1.6788	1.6768	1.6714
140	1.6396	1.6539	1.6649	1.6726	1.6771	1.6783	1.6764	1.6714
150	1.6417	1.6550	1.6653	1.6725	1.6767	1.6779	1.6761	1.6713
160	1.6435	1.6561	1.6657	1.6725	1.6764	1.6775	1.6758	1.6713
170	1.6451	1.6570	1.6661	1.6724	1.6761	1.6772	1.6755	1.6713
180	1.6466	1.6578	1.6664	1.6724	1.6759	1.6768	1.6753	1.6713
190	1.6479	1.6585	1.6666	1.6724	1.6757	1.6766	1.6751	1.6713
200	1.6491	1.6591	1.6669	1.6723	1.6755	1.6763	1.6749	1.6713

Table 10-20 Equations for Calculating Pin Size for Helical Gears in the Normal System

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3\beta}$	$m_n = 1$ $\alpha_n = 20°$ $z = 20$ $\beta = 15°\,00'\,00''$ $x_n = +0.4$ $z_v = 22.19211$ $\frac{\psi_v}{2} = 0.0427566$ $\alpha_v = 24.90647°$ $\phi_v = 0.507078$ $d_p = 1.9020$
2	Half Tooth Space Angle at Base Circle	$\frac{\psi_v}{2}$	$\frac{\pi}{2z_v} - \text{inv}\alpha_n - \frac{2x_n\tan\alpha_n}{z_v}$	
3	Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_v	$\cos^{-1}\left(\frac{z_v\cos\alpha_n}{z_v + 2x_n}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \frac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_n \cos\alpha_n\left(\text{inv}\phi_v + \frac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v are radians.

Table 10-21 Equations for Calculating Over Pins Measurement for Helical Gears in the Normal System

No.	Item	Symbol	Formula	Example
1	Actual Pin Diameter	d_p	See **NOTE**	Let d_p = 2, then
2	Involute Function ϕ	invϕ	$\frac{d_p}{m_n z \cos\alpha_n} - \frac{\pi}{2z} + \mathrm{inv}\alpha_t + \frac{2x_n \tan\alpha_n}{z}$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	α_t =20.646896° invϕ = 0.058890 ϕ = 30.8534 d_m = 24.5696
4	Over Pins Measurement	d_m	Even Teeth: $\frac{z m_n \cos\alpha_t}{\cos\beta\cos\phi} + d_p$ Odd Teeth: $\frac{z m_n \cos\alpha_t}{\cos\beta\cos\phi} \cos\frac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of **Table 10-20**, or its approximate value, is entered as the actual diameter of d_p.

Table 10-22 Equations for Calculating Pin Size for Helical Gears in the Radial System

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\frac{z}{\cos^3\beta}$	m_t = 3 α_t = 20° z = 36 β = 33° 33' 26.3"
2	Half Tooth Space Angle at Base Circle	$\frac{\psi_v}{2}$	$\frac{\pi}{2z_v} - \mathrm{inv}\alpha_n - \frac{2x_t \tan\alpha_t}{z_v}$	α_n = 16.87300° x_t = + 0.2
3	Pressure Angle at the Point Pin is tangent to Tooth Surface	α_v	$\cos^{-1}\left(\frac{z_v \cos\alpha_n}{z_v + 2\frac{x_t}{\cos\beta}}\right)$	z_v = 62.20800 $\frac{\psi_v}{2}$ = 0.014091
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \frac{\psi_v}{2}$	α_v = 18.26390 ϕ_v = 0.34411
5	Ideal Pin Diameter	d_p	$z_v m_t \cos\beta \cos\alpha_n \left(\mathrm{inv}\phi_v + \frac{\psi_v}{2}\right)$	invϕ_v = 0.014258 d_p = 4.2190

NOTE: The units of angles $\psi_v / 2$ and ϕ_v are radians.

Table 10-23 Equations for Calculating Over Pins Measurement for Helical Gears in the Radial System

No.	Item	Symbol	Formula	Example
1	Actual Pin Diameter	d_p	See **NOTE**	
2	Involute Function ϕ	invϕ	$\frac{d_p}{m_t z \cos\beta\cos\alpha_n} - \frac{\pi}{2z} + \mathrm{inv}\alpha_t + \frac{2x_t \tan\alpha_t}{z}$	d_p = 4.2190 invϕ = 0.024302
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	ϕ = 23.3910 d_m = 114.793
4	Over Pins Measurement	d_m	Even Teeth: $\frac{z m_t \cos\alpha_t}{\cos\phi} + d_p$ Odd Teeth: $\frac{z m_t \cos\alpha_t}{\cos\phi} \cos\frac{90°}{z} + d_p$	

NOTE: The ideal pin diameter of **Table 10-22**, or its approximate value, is applied as the actual diameter of pin d_p here.

Table 10-24 Equations for Three Wire Method of Worm Measurement, (a)-1

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\frac{\pi m_x}{2\cos\alpha_x}$	m_x = 2 α_n = 20° z_w = 1 d_1 = 31 γ = 3.691386° α_x = 20.03827°
2	Three Wire Measurement	d_m	$d_1 - \frac{\pi m_x}{2\tan\alpha_x} + d_p\left(1 + \frac{1}{\sin\alpha_x}\right)$	d'_p = 3.3440; let d_p = 3.3 d_m = 35.3173

Table 10-25 Equations for Three Wire Method of Worm Measurement, (a)-2

No.	Item	Symbol	Formula	Example
1	Ideal Pin Diameter	d_p'	$\frac{\pi m_n}{2\cos\alpha_n}$	m_x = 2 α_n = 20° z_w = 1 d_1 = 31
2	Three Wire Measurement	d_m	$d_1 - \frac{\pi m_n}{2\tan\alpha_n} + d_p\left(1 + \frac{1}{\sin\alpha_n}\right) - \frac{(d_p \cos\alpha_n \sin\gamma)^2}{2d_1}$	γ = 3.691386° m_n = 1.99585 d_p' = 3.3363; let d_p = 3.3 d_m = 35.3344

Table 10-26 Equation for Calculating Pin Size for Worms in the Axial System, (b)-1

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z_w}{\cos^3(90-\gamma)}$	$m_x = 2$ $\alpha_n = 20°$ $z_w = 1$ $d_1 = 31$ $\gamma = 3.691386°$ $z_v = 3747.1491$ $\dfrac{\psi_v}{2} = -0.014485$ $\alpha_v = 20°$ $\phi_v = 0.349485$ $\mathrm{inv}\phi_v = 0.014960$ $d_p = 3.3382$
2	Half Tooth Space Angle at Base Circle	$\dfrac{\psi_v}{2}$	$\dfrac{\pi}{2z_v} - \mathrm{inv}\alpha_n$	
3	Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \dfrac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_x \cos\gamma \cos\alpha_n \left(\mathrm{inv}\phi_v + \dfrac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v are radians.

Table 10-27 Equation for Three Wire Method for Worms in the Axial System, (b)-2

No.	Item	Symbol	Formula	Example
1	Actual Pin Size	d_p	See **NOTE 1**	Let $d_p = 3.3$ $\alpha_t = 79.96878°$ $\mathrm{inv}\alpha_t = 4.257549$ $\mathrm{inv}\phi = 4.446297$ $\phi = 80.2959°$ $d_m = 35.3345$
2	Involute Function ϕ	$\mathrm{inv}\phi$	$\dfrac{d_p}{m_x z_w \cos\gamma \cos\alpha_n} - \dfrac{\pi}{2z_w} + \mathrm{inv}\alpha_t$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Three Wire Measurement	d_m	$\dfrac{z_w m_x \cos\alpha_t}{\tan\gamma \cos\phi} + d_p$	

NOTE: 1. The value of ideal pin diameter from **Table 10-26,** or its approximate value, is to be used as the actual pin diameter, d_p.

2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

Table 10-28 Equation for Calculating Pin Size for Worms in the Normal System, (b)-3

No.	Item	Symbol	Formula	Example
1	Number of Teeth of an Equivalent Spur Gear	z_v	$\dfrac{z_w}{\cos^3(90-\gamma)}$	$m_n = 2.5$ $\alpha_n = 20°$ $z_w = 1$ $d_1 = 37$ $\gamma = 3.874288°$ $z_v = 3241.792$ $\dfrac{\psi_v}{2} = -0.014420$ $\alpha_v = 20°$ $\phi_v = 0.349550$ $\mathrm{inv}\phi_v = 0.0149687$ $d_p = 4.1785$
2	Half of Tooth Space Angle at Base Circle	$\dfrac{\psi_v}{2}$	$\dfrac{\pi}{2z_v} - \mathrm{inv}\alpha_n$	
3	Pressure Angle at the Point Pin is Tangent to Tooth Surface	α_v	$\cos^{-1}\left(\dfrac{z_v \cos\alpha_n}{z_v}\right)$	
4	Pressure Angle at Pin Center	ϕ_v	$\tan\alpha_v + \dfrac{\psi_v}{2}$	
5	Ideal Pin Diameter	d_p	$z_v m_n \cos\alpha_n \left(\mathrm{inv}\phi_v + \dfrac{\psi_v}{2}\right)$	

NOTE: The units of angles $\psi_v/2$ and ϕ_v are radians.

Table 10-29 Equations for Three Wire Method for Worms in the Normal System, (b)-4

No.	Item	Symbol	Formula	Example
1	Actual Pin Size	d_p	See **NOTE 1**	$d_p = 4.2$ $\alpha_t = 79.48331°$ $\mathrm{inv}\alpha_t = 3.999514$ $\mathrm{inv}\phi = 4.216536$ $\phi = 79.8947°$ $d_m = 42.6897$
2	Involute Function ϕ	$\mathrm{inv}\phi$	$\dfrac{d_p}{m_n z_w \cos\alpha_n} - \dfrac{\pi}{2z_w} + \mathrm{inv}\alpha_t$	
3	Pressure Angle at Pin Center	ϕ	Find from Involute Function Table	
4	Three Wire Measurement	d_m	$\dfrac{z_w m_n \cos\alpha_t}{\sin\gamma \cos\phi} + d_p$	

NOTE: 1. The value of ideal pin diameter from **Table 10-28**, or its approximate value, is to be used as the actual pin diameter, d_p.

2. $\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$

TABLE 10-30 METRIC GEAR OVER PINS MEASUREMENT
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.30 Wire Size = 0.5184mm; 0.0204 Inch				Module 0.40 Wire Size = 0.6912mm; 0.0272 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
5	1.500	0.0591			2.000	0.0787			**5**
6	1.800	0.0709			2.400	0.0945			**6**
7	2.100	0.0827			2.800	0.1102			**7**
8	2.400	0.0945			3.200	0.1260			**8**
9	2.700	0.1063			3.600	0.1417			**9**
10	3.000	0.1181			4.000	0.1575			**10**
11	3.300	0.1299			4.400	0.1732			**11**
12	3.600	0.1417			4.800	0.1890			**12**
13	3.900	0.1535			5.200	0.2047			**13**
14	4.200	0.1654			5.600	0.2205			**14**
15	4.500	0.1772			6.000	0.2362			**15**
16	4.800	0.1890			6.400	0.2520			**16**
17	5.100	0.2008			6.800	0.2677			**17**
18	5.400	0.2126	6.115	0.2408	7.200	0.2835	8.154	0.3210	**18**
19	5.700	0.2244	6.396	0.2518	7.600	0.2992	8.528	0.3357	**19**
20	6.000	0.2362	6.717	0.2644	8.000	0.3150	8.956	0.3526	**20**
21	6.300	0.2480	7.000	0.2756	8.400	0.3307	9.333	0.3674	**21**
22	6.600	0.2598	7.319	0.2881	8.800	0.3465	9.758	0.3842	**22**
23	6.900	0.2717	7.603	0.2993	9.200	0.3622	10.137	0.3991	**23**
24	7.200	0.2835	7.920	0.3118	9.600	0.3780	10.560	0.4157	**24**
25	7.500	0.2953	8.205	0.3230	10.000	0.3937	10.940	0.4307	**25**
26	7.800	0.3071	8.521	0.3355	10.400	0.4094	11.361	0.4473	**26**
27	8.100	0.3189	8.808	0.3468	10.800	0.4252	11.743	0.4623	**27**
28	8.400	0.3307	9.122	0.3591	11.200	0.4409	12.163	0.4789	**28**
29	8.700	0.3425	9.410	0.3705	11.600	0.4567	12.546	0.4939	**29**
30	9.000	0.3543	9.723	0.3828	12.000	0.4724	12.964	0.5104	**30**
31	9.300	0.3661	10.011	0.3941	12.400	0.4882	13.348	0.5255	**31**
32	9.600	0.3780	10.324	0.4065	12.800	0.5039	13.765	0.5419	**32**
33	9.900	0.3898	10.613	0.4178	13.200	0.5197	14.150	0.5571	**33**
34	10.200	0.4016	10.925	0.4301	13.600	0.5354	14.566	0.5735	**34**
35	10.500	0.4134	11.214	0.4415	14.000	0.5512	14.952	0.5887	**35**
36	10.800	0.4252	11.525	0.4538	14.400	0.5669	15.367	0.6050	**36**
37	11.100	0.4370	11.815	0.4652	14.800	0.5827	15.754	0.6202	**37**
38	11.400	0.4488	12.126	0.4774	15.200	0.5984	16.168	0.6365	**38**
39	11.700	0.4606	12.417	0.4888	15.600	0.6142	16.555	0.6518	**39**
40	12.000	0.4724	12.727	0.5010	16.000	0.6299	16.969	0.6681	**40**
41	12.300	0.4843	13.018	0.5125	16.400	0.6457	17.357	0.6833	**41**
42	12.600	0.4961	13.327	0.5247	16.800	0.6614	17.769	0.6996	**42**
43	12.900	0.5079	13.619	0.5362	17.200	0.6772	18.158	0.7149	**43**
44	13.200	0.5197	13.927	0.5483	17.600	0.6929	18.570	0.7311	**44**
45	13.500	0.5315	14.219	0.5598	18.000	0.7087	18.959	0.7464	**45**
46	13.800	0.5433	14.528	0.5720	18.400	0.7244	19.371	0.7626	**46**
47	14.100	0.5551	14.820	0.5835	18.800	0.7402	19.760	0.7780	**47**
48	14.400	0.5669	15.128	0.5956	19.200	0.7559	20.171	0.7941	**48**
49	14.700	0.5787	15.421	0.6071	19.600	0.7717	20.561	0.8095	**49**
50	15.000	0.5906	15.729	0.6192	20.000	0.7874	20.972	0.8257	**50**
51	15.300	0.6024	16.022	0.6308	20.400	0.8031	21.362	0.8410	**51**
52	15.600	0.6142	16.329	0.6429	20.800	0.8189	21.772	0.8572	**52**
53	15.900	0.6260	16.622	0.6544	21.200	0.8346	22.163	0.8726	**53**
54	16.200	0.6378	16.929	0.6665	21.600	0.8504	22.573	0.8887	**54**
55	16.500	0.6496	17.223	0.6781	22.000	0.8661	22.964	0.9041	**55**
56	16.800	0.6614	17.530	0.6901	22.400	0.8819	23.373	0.9202	**56**
57	17.100	0.6732	17.823	0.7017	22.800	0.8976	23.764	0.9356	**57**
58	17.400	0.6850	18.130	0.7138	23.200	0.9134	24.173	0.9517	**58**
59	17.700	0.6969	18.424	0.7253	23.600	0.9291	24.565	0.9671	**59**
60	18.000	0.7087	18.730	0.7374	24.000	0.9449	24.974	0.9832	**60**
61	18.300	0.7205	19.024	0.7490	24.400	0.9606	25.366	0.9987	**61**
62	18.600	0.7323	19.331	0.7610	24.800	0.9764	25.774	1.0147	**62**
63	18.900	0.7441	19.625	0.7726	25.200	0.9921	26.166	1.0302	**63**
64	19.200	0.7559	19.931	0.7847	25.600	1.0079	26.574	1.0462	**64**
65	19.500	0.7677	20.225	0.7963	26.000	1.0236	26.967	1.0617	**65**
66	19.800	0.7795	20.531	0.8083	26.400	1.0394	27.375	1.0777	**66**
67	20.100	0.7913	20.826	0.8199	26.800	1.0551	27.767	1.0932	**67**
68	20.400	0.8031	21.131	0.8319	27.200	1.0709	28.175	1.1093	**68**
69	20.700	0.8150	21.426	0.8435	27.600	1.0866	28.568	1.1247	**69**
70	21.000	0.8268	21.731	0.8556	28.000	1.1024	28.975	1.1408	**70**
71	21.300	0.8386	22.026	0.8672	28.400	1.1181	29.368	1.1562	**71**
72	21.600	0.8504	22.332	0.8792	28.800	1.1339	29.776	1.1723	**72**
73	21.900	0.8622	22.627	0.8908	29.200	1.1496	30.169	1.1877	**73**
74	22.200	0.8740	22.932	0.9028	29.600	1.1654	30.576	1.2038	**74**
75	22.500	0.8858	23.227	0.9144	30.000	1.1811	30.969	1.2193	**75**
76	22.800	0.8976	23.532	0.9265	30.400	1.1969	31.376	1.2353	**76**
77	23.100	0.9094	23.827	0.9381	30.800	1.2126	31.770	1.2508	**77**
78	23.400	0.9213	24.132	0.9501	31.200	1.2283	32.176	1.2668	**78**
79	23.700	0.9331	24.428	0.9617	31.600	1.2441	32.570	1.2823	**79**
80	24.000	0.9449	24.732	0.9737	32.000	1.2598	32.977	1.2983	**80**
81	24.300	0.9567	25.028	0.9853	32.400	1.2756	33.370	1.3138	**81**
82	24.600	0.9685	25.333	0.9973	32.800	1.2913	33.777	1.3298	**82**
83	24.900	0.9803	25.628	1.0090	33.200	1.3071	34.171	1.3453	**83**
84	25.200	0.9921	25.933	1.0210	33.600	1.3228	34.577	1.3613	**84**
85	25.500	1.0039	26.228	1.0326	34.000	1.3386	34.971	1.3768	**85**
86	25.800	1.0157	26.533	1.0446	34.400	1.3543	35.377	1.3928	**86**
87	26.100	1.0276	26.829	1.0562	34.800	1.3701	35.771	1.4083	**87**
88	26.400	1.0394	27.133	1.0682	35.200	1.3858	36.177	1.4243	**88**
89	26.700	1.0512	27.429	1.0799	35.600	1.4016	36.572	1.4398	**89**
90	27.000	1.0630	27.733	1.0919	36.000	1.4173	36.977	1.4558	**90**
91	27.300	1.0748	28.029	1.1035	36.400	1.4331	37.372	1.4713	**91**
92	27.600	1.0866	28.333	1.1155	36.800	1.4488	37.778	1.4873	**92**
93	27.900	1.0984	28.629	1.1271	37.200	1.4646	38.172	1.5029	**93**
94	28.200	1.1102	28.933	1.1391	37.600	1.4803	38.578	1.5188	**94**
95	28.500	1.1220	29.230	1.1508	38.000	1.4961	38.973	1.5344	**95**
96	28.800	1.1339	29.533	1.1627	38.400	1.5118	39.378	1.5503	**96**
97	29.100	1.1457	29.830	1.1744	38.800	1.5276	39.773	1.5659	**97**
98	29.400	1.1575	30.134	1.1864	39.200	1.5433	40.178	1.5818	**98**
99	29.700	1.1693	30.430	1.1980	39.600	1.5591	40.573	1.5974	**99**
100	30.000	1.1811	30.734	1.2100	40.000	1.5748	40.978	1.6133	**100**
101	30.300	1.1929	31.030	1.2217	40.400	1.5906	41.373	1.6289	**101**
102	30.600	1.2047	31.334	1.2336	40.800	1.6063	41.778	1.6448	**102**
103	30.900	1.2165	31.630	1.2453	41.200	1.6220	42.174	1.6604	**103**
104	31.200	1.2283	31.934	1.2572	41.600	1.6378	42.579	1.6763	**104**
105	31.500	1.2402	32.230	1.2689	42.000	1.6535	42.974	1.6919	**105**
106	31.800	1.2520	32.534	1.2809	42.400	1.6693	43.379	1.7078	**106**
107	32.100	1.2638	32.831	1.2925	42.800	1.6850	43.774	1.7234	**107**
108	32.400	1.2756	33.134	1.3045	43.200	1.7008	44.179	1.7393	**108**
109	32.700	1.2874	33.431	1.3162	43.600	1.7165	44.574	1.7549	**109**

Continued on following page

TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.30 Wire Size = 0.5184mm; 0.0204 Inch				Module 0.40 Wire Size = 0.6912mm; 0.0272 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
110	33.000	1.2992	33.734	1.3281	44.000	1.7323	44.979	1.7708	110
111	33.300	1.3110	34.031	1.3398	44.400	1.7480	45.374	1.7864	111
112	33.600	1.3228	34.334	1.3517	44.800	1.7638	45.779	1.8023	112
113	33.900	1.3346	34.631	1.3634	45.200	1.7795	46.175	1.8179	113
114	34.200	1.3465	34.934	1.3754	45.600	1.7953	46.579	1.8338	114
115	34.500	1.3583	35.231	1.3871	46.000	1.8110	46.975	1.8494	115
116	34.800	1.3701	35.534	1.3990	46.400	1.8268	47.379	1.8653	116
117	35.100	1.3819	35.831	1.4107	46.800	1.8425	47.775	1.8809	117
118	35.400	1.3937	36.135	1.4226	47.200	1.8583	48.179	1.8968	118
119	35.700	1.4055	36.431	1.4343	47.600	1.8740	48.575	1.9124	119
120	36.000	1.4173	36.735	1.4462	48.000	1.8898	48.979	1.9283	120
121	36.300	1.4291	37.032	1.4579	48.400	1.9055	49.375	1.9439	121
122	36.600	1.4409	37.335	1.4699	48.800	1.9213	49.780	1.9598	122
123	36.900	1.4528	37.632	1.4816	49.200	1.9370	50.176	1.9754	123
124	37.200	1.4646	37.935	1.4935	49.600	1.9528	50.580	1.9913	124
125	37.500	1.4764	38.232	1.5052	50.000	1.9685	50.976	2.0069	125
126	37.800	1.4882	38.535	1.5171	50.400	1.9843	51.380	2.0228	126
127	38.100	1.5000	38.832	1.5288	50.800	2.0000	51.776	2.0384	127
128	38.400	1.5118	39.135	1.5407	51.200	2.0157	52.180	2.0543	128
129	38.700	1.5236	39.432	1.5524	51.600	2.0315	52.576	2.0699	129
130	39.000	1.5354	39.735	1.5644	52.000	2.0472	52.980	2.0858	130
131	39.300	1.5472	40.032	1.5761	52.400	2.0630	53.376	2.1014	131
132	39.600	1.5591	40.335	1.5880	52.800	2.0787	53.780	2.1173	132
133	39.900	1.5709	40.632	1.5997	53.200	2.0945	54.176	2.1329	133
134	40.200	1.5827	40.935	1.6116	53.600	2.1102	54.580	2.1488	134
135	40.500	1.5945	41.232	1.6233	54.000	2.1260	54.976	2.1644	135
136	40.800	1.6063	41.535	1.6352	54.400	2.1417	55.380	2.1803	136
137	41.100	1.6181	41.832	1.6469	54.800	2.1575	55.777	2.1959	137
138	41.400	1.6299	42.135	1.6589	55.200	2.1732	56.180	2.2118	138
139	41.700	1.6417	42.433	1.6706	55.600	2.1890	56.577	2.2274	139
140	42.000	1.6535	42.735	1.6825	56.000	2.2047	56.980	2.2433	140
141	42.300	1.6654	43.033	1.6942	56.400	2.2205	57.377	2.2589	141
142	42.600	1.6772	43.335	1.7061	56.800	2.2362	57.780	2.2748	142
143	42.900	1.6890	43.633	1.7178	57.200	2.2520	58.177	2.2904	143
144	43.200	1.7008	43.935	1.7297	57.600	2.2677	58.580	2.3063	144
145	43.500	1.7126	44.233	1.7414	58.000	2.2835	58.977	2.3219	145
146	43.800	1.7244	44.535	1.7534	58.400	2.2992	59.381	2.3378	146
147	44.100	1.7362	44.833	1.7651	58.800	2.3150	59.777	2.3534	147
148	44.400	1.7480	45.135	1.7770	59.200	2.3307	60.181	2.3693	148
149	44.700	1.7598	45.433	1.7887	59.600	2.3465	60.577	2.3849	149
150	45.000	1.7717	45.735	1.8006	60.000	2.3622	60.981	2.4008	150
151	45.300	1.7835	46.033	1.8123	60.400	2.3780	61.377	2.4164	151
152	45.600	1.7953	46.336	1.8242	60.800	2.3937	61.781	2.4323	152
153	45.900	1.8071	46.633	1.8360	61.200	2.4094	62.178	2.4479	153
154	46.200	1.8189	46.936	1.8479	61.600	2.4252	62.581	2.4638	154
155	46.500	1.8307	47.233	1.8596	62.000	2.4409	62.978	2.4794	155
156	46.800	1.8425	47.536	1.8715	62.400	2.4567	63.381	2.4953	156
157	47.100	1.8543	47.833	1.8832	62.800	2.4724	63.778	2.5109	157
158	47.400	1.8661	48.136	1.8951	63.200	2.4882	64.181	2.5268	158
159	47.700	1.8780	48.433	1.9068	63.600	2.5039	64.578	2.5424	159
160	48.000	1.8898	48.736	1.9187	64.000	2.5197	64.981	2.5583	160
161	48.300	1.9016	49.033	1.9305	64.400	2.5354	65.378	2.5739	161
162	48.600	1.9134	49.336	1.9424	64.800	2.5512	65.781	2.5898	162
163	48.900	1.9252	49.633	1.9541	65.200	2.5669	66.178	2.6054	163
164	49.200	1.9370	49.936	1.9660	65.600	2.5827	66.581	2.6213	164
165	49.500	1.9488	50.234	1.9777	66.000	2.5984	66.978	2.6369	165
166	49.800	1.9606	50.536	1.9896	66.400	2.6142	67.381	2.6528	166
167	50.100	1.9724	50.834	2.0013	66.800	2.6299	67.778	2.6684	167
168	50.400	1.9843	51.136	2.0132	67.200	2.6457	68.181	2.6843	168
169	50.700	1.9961	51.434	2.0249	67.600	2.6614	68.578	2.6999	169
170	51.000	2.0079	51.736	2.0368	68.000	2.6772	68.981	2.7158	170
171	51.300	2.0197	52.034	2.0486	68.400	2.6929	69.378	2.7314	171
172	51.600	2.0315	52.336	2.0605	68.800	2.7087	69.781	2.7473	172
173	51.900	2.0433	52.634	2.0722	69.200	2.7244	70.178	2.7629	173
174	52.200	2.0551	52.936	2.0841	69.600	2.7402	70.581	2.7788	174
175	52.500	2.0669	53.234	2.0958	70.000	2.7559	70.979	2.7944	175
176	52.800	2.0787	53.536	2.1077	70.400	2.7717	71.381	2.8103	176
177	53.100	2.0906	53.834	2.1194	70.800	2.7874	71.779	2.8259	177
178	53.400	2.1024	54.136	2.1313	71.200	2.8031	72.181	2.8418	178
179	53.700	2.1142	54.434	2.1431	71.600	2.8189	72.579	2.8574	179
180	54.000	2.1260	54.736	2.1550	72.000	2.8346	72.981	2.8733	180
181	54.300	2.1378	55.034	2.1667	72.400	2.8504	73.379	2.8889	181
182	54.600	2.1496	55.336	2.1786	72.800	2.8661	73.782	2.9048	182
183	54.900	2.1614	55.634	2.1903	73.200	2.8819	74.179	2.9204	183
184	55.200	2.1732	55.936	2.2022	73.600	2.8976	74.582	2.9363	184
185	55.500	2.1850	56.234	2.2139	74.000	2.9134	74.979	2.9519	185
186	55.800	2.1969	56.536	2.2258	74.400	2.9291	75.382	2.9678	186
187	56.100	2.2087	56.834	2.2376	74.800	2.9449	75.779	2.9834	187
188	56.400	2.2205	57.136	2.2495	75.200	2.9606	76.182	2.9993	188
189	56.700	2.2323	57.434	2.2612	75.600	2.9764	76.579	3.0149	189
190	57.000	2.2441	57.736	2.2731	76.000	2.9921	76.982	3.0308	190
191	57.300	2.2559	58.036	2.2849	76.400	3.0079	77.382	3.0465	191
192	57.600	2.2677	58.336	2.2967	76.800	3.0236	77.782	3.0623	192
193	57.900	2.2795	58.636	2.3085	77.200	3.0394	78.182	3.0780	193
194	58.200	2.2913	58.936	2.3203	77.600	3.0551	78.582	3.0938	194
195	58.500	2.3031	59.236	2.3321	78.000	3.0709	78.982	3.1095	195
196	58.800	2.3150	59.536	2.3440	78.400	3.0866	79.382	3.1253	196
197	59.100	2.3268	59.836	2.3558	78.800	3.1024	79.782	3.1410	197
198	59.400	2.3386	60.136	2.3676	79.200	3.1181	80.182	3.1568	198
199	59.700	2.3504	60.436	2.3794	79.600	3.1339	80.582	3.1725	199
200	60.000	2.3622	60.736	2.3912	80.000	3.1496	80.982	3.1883	200
201	60.300	2.3740	61.035	2.4029	80.400	3.1654	81.379	3.2039	201
202	60.600	2.3858	61.335	2.4147	80.800	3.1811	81.780	3.2197	202
203	60.900	2.3976	61.635	2.4266	81.200	3.1969	82.180	3.2354	203
204	61.200	2.4094	61.935	2.4384	81.600	3.2126	82.580	3.2512	204
205	61.500	2.4213	62.235	2.4502	82.000	3.2283	82.980	3.2669	205
240	72.000	2.8346	72.737	2.8637	96.000	3.7795	96.982	3.8182	240
280	84.000	3.3071	84.737	3.3361	112.000	4.4094	112.983	4.4481	280
300	90.000	3.5433	90.737	3.5723	120.000	4.7244	120.983	4.7631	300
340	102.000	4.0157	102.738	4.0448	136.000	5.3543	136.983	5.3930	340
380	114.000	4.4882	114.738	4.5172	152.000	5.9843	152.984	6.0230	380
400	120.000	4.7244	120.738	4.7535	160.000	6.2992	160.984	6.3379	400
440	132.000	5.1969	132.738	5.2259	176.000	6.9291	176.984	6.9679	440
480	144.000	5.6693	144.738	5.6984	192.000	7.5591	192.984	7.5978	480
500	150.000	5.9055	150.738	5.9346	200.000	7.8740	200.984	7.9128	500

Continued on following page

TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.50 Wire Size = 0.8640mm; 0.0340 Inch				Module 0.75 Wire Size = 1.2960mm; 0.0510 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
5	2.500	0.0984			3.750	0.1476			**5**
6	3.000	0.1181			4.500	0.1772			**6**
7	3.500	0.1378			5.250	0.2067			**7**
8	4.000	0.1575			6.000	0.2362			**8**
9	4.500	0.1772			6.750	0.2657			**9**
10	5.000	0.1969			7.500	0.2953			**10**
11	5.500	0.2165			8.250	0.3248			**11**
12	6.000	0.2362			9.000	0.3543			**12**
13	6.500	0.2559			9.750	0.3839			**13**
14	7.000	0.2756			10.500	0.4134			**14**
15	7.500	0.2953			11.250	0.4429			**15**
16	8.000	0.3150			12.000	0.4724			**16**
17	8.500	0.3346			12.750	0.5020			**17**
18	9.000	0.3543	10.192	0.4013	13.500	0.5315	15.288	0.6019	**18**
19	9.500	0.3740	10.660	0.4197	14.250	0.5610	15.990	0.6295	**19**
20	10.000	0.3937	11.195	0.4407	15.000	0.5906	16.792	0.6611	**20**
21	10.500	0.4134	11.666	0.4593	15.750	0.6201	17.499	0.6889	**21**
22	11.000	0.4331	12.198	0.4802	16.500	0.6496	18.296	0.7203	**22**
23	11.500	0.4528	12.671	0.4989	17.250	0.6791	19.007	0.7483	**23**
24	12.000	0.4724	13.200	0.5197	18.000	0.7087	19.800	0.7795	**24**
25	12.500	0.4921	13.676	0.5384	18.750	0.7382	20.513	0.8076	**25**
26	13.000	0.5118	14.202	0.5591	19.500	0.7677	21.303	0.8387	**26**
27	13.500	0.5315	14.679	0.5779	20.250	0.7972	22.019	0.8669	**27**
28	14.000	0.5512	15.204	0.5986	21.000	0.8268	22.805	0.8978	**28**
29	14.500	0.5709	15.683	0.6174	21.750	0.8563	23.524	0.9261	**29**
30	15.000	0.5906	16.205	0.6380	22.500	0.8858	24.308	0.9570	**30**
31	15.500	0.6102	16.685	0.6569	23.250	0.9154	25.028	0.9854	**31**
32	16.000	0.6299	17.206	0.6774	24.000	0.9449	25.810	1.0161	**32**
33	16.500	0.6496	17.688	0.6964	24.750	0.9744	26.532	1.0446	**33**
34	17.000	0.6693	18.208	0.7168	25.500	1.0039	27.312	1.0753	**34**
35	17.500	0.6890	18.690	0.7358	26.250	1.0335	28.036	1.1038	**35**
36	18.000	0.7087	19.209	0.7563	27.000	1.0630	28.813	1.1344	**36**
37	18.500	0.7283	19.692	0.7753	27.750	1.0925	29.539	1.1629	**37**
38	19.000	0.7480	20.210	0.7957	28.500	1.1220	30.315	1.1935	**38**
39	19.500	0.7677	20.694	0.8147	29.250	1.1516	31.041	1.2221	**39**
40	20.000	0.7874	21.211	0.8351	30.000	1.1811	31.816	1.2526	**40**
41	20.500	0.8071	21.696	0.8542	30.750	1.2106	32.544	1.2813	**41**
42	21.000	0.8268	22.212	0.8745	31.500	1.2402	33.318	1.3117	**42**
43	21.500	0.8465	22.698	0.8936	32.250	1.2697	34.046	1.3404	**43**
44	22.000	0.8661	23.212	0.9139	33.000	1.2992	34.819	1.3708	**44**
45	22.500	0.8858	23.699	0.9330	33.750	1.3287	35.548	1.3995	**45**
46	23.000	0.9055	24.213	0.9533	34.500	1.3583	36.320	1.4299	**46**
47	23.500	0.9252	24.700	0.9725	35.250	1.3878	37.051	1.4587	**47**
48	24.000	0.9449	25.214	0.9927	36.000	1.4173	37.821	1.4890	**48**
49	24.500	0.9646	25.702	1.0119	36.750	1.4469	38.552	1.5178	**49**
50	25.000	0.9843	26.215	1.0321	37.500	1.4764	39.322	1.5481	**50**
51	25.500	1.0039	26.703	1.0513	38.250	1.5059	40.054	1.5769	**51**
52	26.000	1.0236	27.215	1.0715	39.000	1.5354	40.823	1.6072	**52**
53	26.500	1.0433	27.704	1.0907	39.750	1.5650	41.556	1.6360	**53**
54	27.000	1.0630	28.216	1.1109	40.500	1.5945	42.324	1.6663	**54**
55	27.500	1.0827	28.705	1.1301	41.250	1.6240	43.057	1.6952	**55**
56	28.000	1.1024	29.216	1.1502	42.000	1.6535	43.824	1.7254	**56**
57	28.500	1.1220	29.706	1.1695	42.750	1.6831	44.558	1.7543	**57**
58	29.000	1.1417	30.217	1.1896	43.500	1.7126	45.325	1.7845	**58**
59	29.500	1.1614	30.706	1.2089	44.250	1.7421	46.060	1.8134	**59**
60	30.000	1.1811	31.217	1.2290	45.000	1.7717	46.826	1.8435	**60**
61	30.500	1.2008	31.707	1.2483	45.750	1.8012	47.561	1.8725	**61**
62	31.000	1.2205	32.218	1.2684	46.500	1.8307	48.326	1.9026	**62**
63	31.500	1.2402	32.708	1.2877	47.250	1.8602	49.062	1.9316	**63**
64	32.000	1.2598	33.218	1.3078	48.000	1.8898	49.827	1.9617	**64**
65	32.500	1.2795	33.709	1.3271	48.750	1.9193	50.563	1.9907	**65**
66	33.000	1.2992	34.218	1.3472	49.500	1.9488	51.328	2.0208	**66**
67	33.500	1.3189	34.709	1.3665	50.250	1.9783	52.064	2.0498	**67**
68	34.000	1.3386	35.219	1.3866	51.000	2.0079	52.828	2.0799	**68**
69	34.500	1.3583	35.710	1.4059	51.750	2.0374	53.565	2.1089	**69**
70	35.000	1.3780	36.219	1.4260	52.500	2.0669	54.329	2.1389	**70**
71	35.500	1.3976	36.710	1.4453	53.250	2.0965	55.066	2.1679	**71**
72	36.000	1.4173	37.219	1.4653	54.000	2.1260	55.829	2.1980	**72**
73	36.500	1.4370	37.711	1.4847	54.750	2.1555	56.567	2.2270	**73**
74	37.000	1.4567	38.220	1.5047	55.500	2.1850	57.330	2.2571	**74**
75	37.500	1.4764	38.712	1.5241	56.250	2.2146	58.067	2.2861	**75**
76	38.000	1.4961	39.220	1.5441	57.000	2.2441	58.830	2.3161	**76**
77	38.500	1.5157	39.712	1.5635	57.750	2.2736	59.568	2.3452	**77**
78	39.000	1.5354	40.220	1.5835	58.500	2.3031	60.331	2.3752	**78**
79	39.500	1.5551	40.713	1.6029	59.250	2.3327	61.069	2.4043	**79**
80	40.000	1.5748	41.221	1.6229	60.000	2.3622	61.831	2.4343	**80**
81	40.500	1.5945	41.713	1.6422	60.750	2.3917	62.570	2.4634	**81**
82	41.000	1.6142	42.221	1.6622	61.500	2.4213	63.331	2.4934	**82**
83	41.500	1.6339	42.714	1.6816	62.250	2.4508	64.070	2.5225	**83**
84	42.000	1.6535	43.221	1.7016	63.000	2.4803	64.832	2.5524	**84**
85	42.500	1.6732	43.714	1.7210	63.750	2.5098	65.571	2.5815	**85**
86	43.000	1.6929	44.221	1.7410	64.500	2.5394	66.332	2.6115	**86**
87	43.500	1.7126	44.714	1.7604	65.250	2.5689	67.072	2.6406	**87**
88	44.000	1.7323	45.222	1.7804	66.000	2.5984	67.832	2.6706	**88**
89	44.500	1.7520	45.715	1.7998	66.750	2.6280	68.572	2.6997	**89**
90	45.000	1.7717	46.222	1.8198	67.500	2.6575	69.333	2.7296	**90**
91	45.500	1.7913	46.715	1.8392	68.250	2.6870	70.073	2.7588	**91**
92	46.000	1.8110	47.222	1.8591	69.000	2.7165	70.833	2.7887	**92**
93	46.500	1.8307	47.715	1.8786	69.750	2.7461	71.573	2.8178	**93**
94	47.000	1.8504	48.222	1.8985	70.500	2.7756	72.333	2.8478	**94**
95	47.500	1.8701	48.716	1.9179	71.250	2.8051	73.074	2.8769	**95**
96	48.000	1.8898	49.222	1.9379	72.000	2.8346	73.834	2.9068	**96**
97	48.500	1.9094	49.716	1.9573	72.750	2.8642	74.574	2.9360	**97**
98	49.000	1.9291	50.223	1.9773	73.500	2.8937	75.334	2.9659	**98**
99	49.500	1.9488	50.716	1.9967	74.250	2.9232	76.075	2.9951	**99**
100	50.000	1.9685	51.223	2.0166	75.000	2.9528	76.834	3.0250	**100**
101	50.500	1.9882	51.717	2.0361	75.750	2.9823	77.575	3.0541	**101**
102	51.000	2.0079	52.223	2.0560	76.500	3.0118	78.334	3.0840	**102**
103	51.500	2.0276	52.717	2.0755	77.250	3.0413	79.076	3.1132	**103**
104	52.000	2.0472	53.223	2.0954	78.000	3.0709	79.835	3.1431	**104**
105	52.500	2.0669	53.717	2.1149	78.750	3.1004	80.576	3.1723	**105**
106	53.000	2.0866	54.223	2.1348	79.500	3.1299	81.335	3.2022	**106**
107	53.500	2.1063	54.718	2.1542	80.250	3.1594	82.076	3.2314	**107**
108	54.000	2.1260	55.223	2.1742	81.000	3.1890	82.835	3.2612	**108**
109	54.500	2.1457	55.718	2.1936	81.750	3.2185	83.577	3.2904	**109**

Continued on following page

TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.50 Wire Size = 0.8640mm; 0.0340 Inch				Module 0.75 Wire Size = 1.2960mm; 0.0510 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
110	55.000	2.1654	56.224	2.2135	82.500	3.2480	84.335	3.3203	**110**
111	55.500	2.1850	56.718	2.2330	83.250	3.2776	85.077	3.3495	**111**
112	56.000	2.2047	57.224	2.2529	84.000	3.3071	85.836	3.3794	**112**
113	56.500	2.2244	57.718	2.2724	84.750	3.3366	86.578	3.4086	**113**
114	57.000	2.2441	58.224	2.2923	85.500	3.3661	87.336	3.4384	**114**
115	57.500	2.2638	58.719	2.3118	86.250	3.3957	88.078	3.4676	**115**
116	58.000	2.2835	59.224	2.3317	87.000	3.4252	88.836	3.4975	**116**
117	58.500	2.3031	59.719	2.3511	87.750	3.4547	89.578	3.5267	**117**
118	59.000	2.3228	60.224	2.3710	88.500	3.4843	90.336	3.5565	**118**
119	59.500	2.3425	60.719	2.3905	89.250	3.5138	91.078	3.5858	**119**
120	60.000	2.3622	61.224	2.4104	90.000	3.5433	91.836	3.6156	**120**
121	60.500	2.3819	61.719	2.4299	90.750	3.5728	92.579	3.6448	**121**
122	61.000	2.4016	62.224	2.4498	91.500	3.6024	93.337	3.6747	**122**
123	61.500	2.4213	62.719	2.4693	92.250	3.6319	94.079	3.7039	**123**
124	62.000	2.4409	63.225	2.4892	93.000	3.6614	94.837	3.7337	**124**
125	62.500	2.4606	63.720	2.5086	93.750	3.6909	95.579	3.7630	**125**
126	63.000	2.4803	64.225	2.5285	94.500	3.7205	96.337	3.7928	**126**
127	63.500	2.5000	64.720	2.5480	95.250	3.7500	97.080	3.8220	**127**
128	64.000	2.5197	65.225	2.5679	96.000	3.7795	97.837	3.8519	**128**
129	64.500	2.5394	65.720	2.5874	96.750	3.8091	98.580	3.8811	**129**
130	65.000	2.5591	66.225	2.6073	97.500	3.8386	99.337	3.9109	**130**
131	65.500	2.5787	66.720	2.6268	98.250	3.8681	100.080	3.9402	**131**
132	66.000	2.5984	67.225	2.6467	99.000	3.8976	100.837	3.9700	**132**
133	66.500	2.6181	67.720	2.6662	99.750	3.9272	101.581	3.9992	**133**
134	67.000	2.6378	68.225	2.6860	100.500	3.9567	102.338	4.0290	**134**
135	67.500	2.6575	68.721	2.7055	101.250	3.9862	103.081	4.0583	**135**
136	68.000	2.6772	69.225	2.7254	102.000	4.0157	103.838	4.0881	**136**
137	68.500	2.6969	69.721	2.7449	102.750	4.0453	104.581	4.1174	**137**
138	69.000	2.7165	70.225	2.7648	103.500	4.0748	105.338	4.1472	**138**
139	69.500	2.7362	70.721	2.7843	104.250	4.1043	106.081	4.1764	**139**
140	70.000	2.7559	71.225	2.8041	105.000	4.1339	106.838	4.2062	**140**
141	70.500	2.7756	71.721	2.8237	105.750	4.1634	107.582	4.2355	**141**
142	71.000	2.7953	72.225	2.8435	106.500	4.1929	108.338	4.2653	**142**
143	71.500	2.8150	72.721	2.8630	107.250	4.2224	109.082	4.2946	**143**
144	72.000	2.8346	73.226	2.8829	108.000	4.2520	109.838	4.3243	**144**
145	72.500	2.8543	73.721	2.9024	108.750	4.2815	110.582	4.3536	**145**
146	73.000	2.8740	74.226	2.9223	109.500	4.3110	111.338	4.3834	**146**
147	73.500	2.8937	74.721	2.9418	110.250	4.3406	112.082	4.4127	**147**
148	74.000	2.9134	75.226	2.9616	111.000	4.3701	112.839	4.4425	**148**
149	74.500	2.9331	75.722	2.9812	111.750	4.3996	113.582	4.4718	**149**
150	75.000	2.9528	76.226	3.0010	112.500	4.4291	114.339	4.5015	**150**
151	75.500	2.9724	76.722	3.0205	113.250	4.4587	115.083	4.5308	**151**
152	76.000	2.9921	77.226	3.0404	114.000	4.4882	115.839	4.5606	**152**
153	76.500	3.0118	77.722	3.0599	114.750	4.5177	116.583	4.5899	**153**
154	77.000	3.0315	78.226	3.0798	115.500	4.5472	117.339	4.6196	**154**
155	77.500	3.0512	78.722	3.0993	116.250	4.5768	118.083	4.6489	**155**
156	78.000	3.0709	79.226	3.1191	117.000	4.6063	118.839	4.6787	**156**
157	78.500	3.0906	79.722	3.1387	117.750	4.6358	119.583	4.7080	**157**
158	79.000	3.1102	80.226	3.1585	118.500	4.6654	120.339	4.7378	**158**
159	79.500	3.1299	80.722	3.1780	119.250	4.6949	121.083	4.7671	**159**
160	80.000	3.1496	81.226	3.1979	120.000	4.7244	121.839	4.7968	**160**
161	80.500	3.1693	81.722	3.2174	120.750	4.7539	122.584	4.8261	**161**
162	81.000	3.1890	82.226	3.2373	121.500	4.7835	123.339	4.8559	**162**
163	81.500	3.2087	82.722	3.2568	122.250	4.8130	124.084	4.8852	**163**
164	82.000	3.2283	83.226	3.2766	123.000	4.8425	124.840	4.9149	**164**
165	82.500	3.2480	83.723	3.2962	123.750	4.8720	125.584	4.9443	**165**
166	83.000	3.2677	84.226	3.3160	124.500	4.9016	126.340	4.9740	**166**
167	83.500	3.2874	84.723	3.3355	125.250	4.9311	127.084	5.0033	**167**
168	84.000	3.3071	85.226	3.3554	126.000	4.9606	127.840	5.0331	**168**
169	84.500	3.3268	85.723	3.3749	126.750	4.9902	128.584	5.0624	**169**
170	85.000	3.3465	86.227	3.3947	127.500	5.0197	129.340	5.0921	**170**
171	85.500	3.3661	86.723	3.4143	128.250	5.0492	130.084	5.1214	**171**
172	86.000	3.3858	87.227	3.4341	129.000	5.0787	130.840	5.1512	**172**
173	86.500	3.4055	87.723	3.4537	129.750	5.1083	131.585	5.1805	**173**
174	87.000	3.4252	88.227	3.4735	130.500	5.1378	132.340	5.2102	**174**
175	87.500	3.4449	88.723	3.4930	131.250	5.1673	133.085	5.2396	**175**
176	88.000	3.4646	89.227	3.5129	132.000	5.1969	133.840	5.2693	**176**
177	88.500	3.4843	89.723	3.5324	132.750	5.2264	134.585	5.2986	**177**
178	89.000	3.5039	90.227	3.5522	133.500	5.2559	135.340	5.3284	**178**
179	89.500	3.5236	90.723	3.5718	134.250	5.2854	136.085	5.3577	**179**
180	90.000	3.5433	91.227	3.5916	135.000	5.3150	136.840	5.3874	**180**
181	90.500	3.5630	91.723	3.6112	135.750	5.3445	137.585	5.4167	**181**
182	91.000	3.5827	92.227	3.6310	136.500	5.3740	138.340	5.4465	**182**
183	91.500	3.6024	92.724	3.6505	137.250	5.4035	139.085	5.4758	**183**
184	92.000	3.6220	93.227	3.6704	138.000	5.4331	139.840	5.5055	**184**
185	92.500	3.6417	93.724	3.6899	138.750	5.4626	140.585	5.5349	**185**
186	93.000	3.6614	94.227	3.7097	139.500	5.4921	141.340	5.5646	**186**
187	93.500	3.6811	94.724	3.7293	140.250	5.5217	142.086	5.5939	**187**
188	94.000	3.7008	95.227	3.7491	141.000	5.5512	142.841	5.6236	**188**
189	94.500	3.7205	95.724	3.7687	141.750	5.5807	143.586	5.6530	**189**
190	95.000	3.7402	96.227	3.7885	142.500	5.6102	144.341	5.6827	**190**
191	95.500	3.7598	96.727	3.8082	143.250	5.6398	145.091	5.7122	**191**
192	96.000	3.7795	97.227	3.8278	144.000	5.6693	145.841	5.7418	**192**
193	96.500	3.7992	97.727	3.8475	144.750	5.6988	146.591	5.7713	**193**
194	97.000	3.8189	98.227	3.8672	145.500	5.7283	147.341	5.8008	**194**
195	97.500	3.8386	98.727	3.8869	146.250	5.7579	148.091	5.8303	**195**
196	98.000	3.8583	99.227	3.9066	147.000	5.7874	148.841	5.8599	**196**
197	98.500	3.8780	99.727	3.9263	147.750	5.8169	149.591	5.8894	**197**
198	99.000	3.8976	100.227	3.9460	148.500	5.8465	150.341	5.9189	**198**
199	99.500	3.9173	100.727	3.9656	149.250	5.8760	151.091	5.9485	**199**
200	100.000	3.9370	101.227	3.9853	150.000	5.9055	151.841	5.9780	**200**
201	100.500	3.9567	101.724	4.0049	150.750	5.9350	152.587	6.0073	**201**
202	101.000	3.9764	102.224	4.0246	151.500	5.9646	153.337	6.0369	**202**
203	101.500	3.9961	102.724	4.0443	152.250	5.9941	154.087	6.0664	**203**
204	102.000	4.0157	103.224	4.0640	153.000	6.0236	154.837	6.0959	**204**
205	102.500	4.0354	103.725	4.0837	153.750	6.0531	155.587	6.1255	**205**
240	120.000	4.7244	121.228	4.7728	180.000	7.0866	181.842	7.1591	**240**
280	140.000	5.5118	141.229	5.5602	210.000	8.2677	211.843	8.3403	**280**
300	150.000	5.9055	151.229	5.9539	225.000	8.8583	226.843	8.9308	**300**
340	170.000	6.6929	171.229	6.7413	255.000	10.0394	256.844	10.1120	**340**
380	190.000	7.4803	191.230	7.5287	285.000	11.2205	286.844	11.2931	**380**
400	200.000	7.8740	201.230	7.9224	300.000	11.8110	301.845	11.8836	**400**
440	220.000	8.6614	221.230	8.7098	330.000	12.9921	331.845	13.0648	**440**
480	240.000	9.4488	241.230	9.4973	360.000	14.1732	361.845	14.2459	**480**
500	250.000	9.8425	251.230	9.8910	375.000	14.7638	376.845	14.8364	**500**

Continued on following page

TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.80 Wire Size = 1.3824mm ; 0.0544 Inch				Module 1.00 Wire Size = 1.7280mm ; 0.0680 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
5	4.000	0.1575			5.000	0.1969			5
6	4.800	0.1890			6.000	0.2362			6
7	5.600	0.2205			7.000	0.2756			7
8	6.400	0.2520			8.000	0.3150			8
9	7.200	0.2835			9.000	0.3543			9
10	8.000	0.3150			10.000	0.3937			10
11	8.800	0.3465			11.000	0.4331			11
12	9.600	0.3780			12.000	0.4724			12
13	10.400	0.4094			13.000	0.5118			13
14	11.200	0.4409			14.000	0.5512			14
15	12.000	0.4724			15.000	0.5906			15
16	12.800	0.5039			16.000	0.6299			16
17	13.600	0.5354			17.000	0.6693			17
18	14.400	0.5669	16.307	0.6420	18.000	0.7087	20.384	0.8025	18
19	15.200	0.5984	17.056	0.6715	19.000	0.7480	21.320	0.8394	19
20	16.000	0.6299	17.912	0.7052	20.000	0.7874	22.390	0.8815	20
21	16.800	0.6614	18.666	0.7349	21.000	0.8268	23.332	0.9186	21
22	17.600	0.6929	19.516	0.7684	22.000	0.8661	24.395	0.9604	22
23	18.400	0.7244	20.274	0.7982	23.000	0.9055	25.342	0.9977	23
24	19.200	0.7559	21.120	0.8315	24.000	0.9449	26.400	1.0394	24
25	20.000	0.7874	21.881	0.8615	25.000	0.9843	27.351	1.0768	25
26	20.800	0.8189	22.723	0.8946	26.000	1.0236	28.404	1.1183	26
27	21.600	0.8504	23.487	0.9247	27.000	1.0630	29.359	1.1559	27
28	22.400	0.8819	24.326	0.9577	28.000	1.1024	30.407	1.1971	28
29	23.200	0.9134	25.092	0.9879	29.000	1.1417	31.365	1.2349	29
30	24.000	0.9449	25.928	1.0208	30.000	1.1811	32.410	1.2760	30
31	24.800	0.9764	26.697	1.0511	31.000	1.2205	33.371	1.3138	31
32	25.600	1.0079	27.530	1.0839	32.000	1.2598	34.413	1.3548	32
33	26.400	1.0394	28.301	1.1142	33.000	1.2992	35.376	1.3928	33
34	27.200	1.0709	29.132	1.1469	34.000	1.3386	36.415	1.4337	34
35	28.000	1.1024	29.905	1.1773	35.000	1.3780	37.381	1.4717	35
36	28.800	1.1339	30.734	1.2100	36.000	1.4173	38.418	1.5125	36
37	29.600	1.1654	31.508	1.2405	37.000	1.4567	39.385	1.5506	37
38	30.400	1.1969	32.336	1.2731	38.000	1.4961	40.420	1.5913	38
39	31.200	1.2283	33.111	1.3036	39.000	1.5354	41.389	1.6295	39
40	32.000	1.2598	33.937	1.3361	40.000	1.5748	42.422	1.6701	40
41	32.800	1.2913	34.714	1.3667	41.000	1.6142	43.392	1.7083	41
42	33.600	1.3228	35.539	1.3992	42.000	1.6535	44.423	1.7490	42
43	34.400	1.3543	36.316	1.4298	43.000	1.6929	45.395	1.7872	43
44	35.200	1.3858	37.140	1.4622	44.000	1.7323	46.425	1.8278	44
45	36.000	1.4173	37.918	1.4929	45.000	1.7717	47.398	1.8661	45
46	36.800	1.4488	38.741	1.5252	46.000	1.8110	48.426	1.9066	46
47	37.600	1.4803	39.521	1.5559	47.000	1.8504	49.401	1.9449	47
48	38.400	1.5118	40.342	1.5883	48.000	1.8898	50.428	1.9854	48
49	39.200	1.5433	41.122	1.6190	49.000	1.9291	51.403	2.0237	49
50	40.000	1.5748	41.943	1.6513	50.000	1.9685	52.429	2.0641	50
51	40.800	1.6063	42.724	1.6821	51.000	2.0079	53.405	2.1026	51
52	41.600	1.6378	43.544	1.7143	52.000	2.0472	54.430	2.1429	52
53	42.400	1.6693	44.326	1.7451	53.000	2.0866	55.407	2.1814	53
54	43.200	1.7008	45.145	1.7774	54.000	2.1260	56.431	2.2217	54
55	44.000	1.7323	45.927	1.8082	55.000	2.1654	57.409	2.2602	55
56	44.800	1.7638	46.746	1.8404	56.000	2.2047	58.432	2.3005	56
57	45.600	1.7953	47.529	1.8712	57.000	2.2441	59.411	2.3390	57
58	46.400	1.8268	48.347	1.9034	58.000	2.2835	60.433	2.3793	58
59	47.200	1.8583	49.130	1.9343	59.000	2.3228	61.413	2.4178	59
60	48.000	1.8898	49.948	1.9664	60.000	2.3622	62.434	2.4580	60
61	48.800	1.9213	50.732	1.9973	61.000	2.4016	63.414	2.4966	61
62	49.600	1.9528	51.548	2.0295	62.000	2.4409	64.435	2.5368	62
63	50.400	1.9843	52.333	2.0603	63.000	2.4803	65.416	2.5754	63
64	51.200	2.0157	53.149	2.0925	64.000	2.5197	66.436	2.6156	64
65	52.000	2.0472	53.934	2.1234	65.000	2.5591	67.417	2.6542	65
66	52.800	2.0787	54.750	2.1555	66.000	2.5984	68.437	2.6944	66
67	53.600	2.1102	55.535	2.1864	67.000	2.6378	69.419	2.7330	67
68	54.400	2.1417	56.350	2.2185	68.000	2.6772	70.438	2.7731	68
69	55.200	2.1732	57.136	2.2494	69.000	2.7165	71.420	2.8118	69
70	56.000	2.2047	57.951	2.2815	70.000	2.7559	72.438	2.8519	70
71	56.800	2.2362	58.737	2.3125	71.000	2.7953	73.421	2.8906	71
72	57.600	2.2677	59.551	2.3445	72.000	2.8346	74.439	2.9307	72
73	58.400	2.2992	60.338	2.3755	73.000	2.8740	75.422	2.9694	73
74	59.200	2.3307	61.152	2.4075	74.000	2.9134	76.440	3.0094	74
75	60.000	2.3622	61.939	2.4385	75.000	2.9528	77.423	3.0482	75
76	60.800	2.3937	62.752	2.4706	76.000	2.9921	78.440	3.0882	76
77	61.600	2.4252	63.539	2.5015	77.000	3.0315	79.424	3.1269	77
78	62.400	2.4567	64.353	2.5336	78.000	3.0709	80.441	3.1670	78
79	63.200	2.4882	65.140	2.5646	79.000	3.1102	81.425	3.2057	79
80	64.000	2.5197	65.953	2.5966	80.000	3.1496	82.441	3.2457	80
81	64.800	2.5512	66.741	2.6276	81.000	3.1890	83.426	3.2845	81
82	65.600	2.5827	67.553	2.6596	82.000	3.2283	84.442	3.3245	82
83	66.400	2.6142	68.342	2.6906	83.000	3.2677	85.427	3.3633	83
84	67.200	2.6457	69.154	2.7226	84.000	3.3071	86.442	3.4032	84
85	68.000	2.6772	69.942	2.7536	85.000	3.3465	87.428	3.4420	85
86	68.800	2.7087	70.754	2.7856	86.000	3.3858	88.443	3.4820	86
87	69.600	2.7402	71.543	2.8167	87.000	3.4252	89.429	3.5208	87
88	70.400	2.7717	72.355	2.8486	88.000	3.4646	90.443	3.5608	88
89	71.200	2.8031	73.144	2.8797	89.000	3.5039	91.429	3.5996	89
90	72.000	2.8346	73.955	2.9116	90.000	3.5433	92.444	3.6395	90
91	72.800	2.8661	74.744	2.9427	91.000	3.5827	93.430	3.6784	91
92	73.600	2.8976	75.555	2.9746	92.000	3.6220	94.444	3.7183	92
93	74.400	2.9291	76.345	3.0057	93.000	3.6614	95.431	3.7571	93
94	75.200	2.9606	77.156	3.0376	94.000	3.7008	96.444	3.7970	94
95	76.000	2.9921	77.945	3.0687	95.000	3.7402	97.432	3.8359	95
96	76.800	3.0236	78.756	3.1006	96.000	3.7795	98.445	3.8758	96
97	77.600	3.0551	79.546	3.1317	97.000	3.8189	99.432	3.9147	97
98	78.400	3.0866	80.356	3.1636	98.000	3.8583	100.445	3.9545	98
99	79.200	3.1181	81.146	3.1947	99.000	3.8976	101.433	3.9934	99
100	80.000	3.1496	81.956	3.2266	100.000	3.9370	102.446	4.0333	100
101	80.800	3.1811	82.747	3.2577	101.000	3.9764	103.433	4.0722	101
102	81.600	3.2126	83.557	3.2896	102.000	4.0157	104.446	4.1120	102
103	82.400	3.2441	84.347	3.3208	103.000	4.0551	105.434	4.1509	103
104	83.200	3.2756	85.157	3.3526	104.000	4.0945	106.446	4.1908	104
105	84.000	3.3071	85.948	3.3838	105.000	4.1339	107.435	4.2297	105
106	84.800	3.3386	86.757	3.4156	106.000	4.1732	108.447	4.2696	106
107	85.600	3.3701	87.548	3.4468	107.000	4.2126	109.435	4.3085	107
108	86.400	3.4016	88.358	3.4786	108.000	4.2520	110.447	4.3483	108
109	87.200	3.4331	89.149	3.5098	109.000	4.2913	111.436	4.3872	109

Continued on following page

TABLE 10-30 (Cont.) **METRIC GEAR OVER PINS MEASUREMENT**
Pitch Diameter and Measurement Over Wires for External, Module Type Gears, 20-Degree Pressure Angle

No. of Teeth	Module 0.80 Wire Size = 1.3824mm ; 0.0544 Inch				Module 1.00 Wire Size = 1.7280mm ; 0.0680 Inch				No. of Teeth
	Pitch Diameter		Meas. Over Wire		Pitch Diameter		Meas. Over Wire		
	mm	Inch	mm	Inch	mm	Inch	mm	Inch	
110	88.000	3.4646	89.958	3.5416	110.000	4.3307	112.447	4.4271	**110**
111	88.800	3.4961	90.749	3.5728	111.000	4.3701	113.436	4.4660	**111**
112	89.600	3.5276	91.558	3.6046	112.000	4.4094	114.447	4.5058	**112**
113	90.400	3.5591	92.349	3.6358	113.000	4.4488	115.437	4.5448	**113**
114	91.200	3.5906	93.158	3.6676	114.000	4.4882	116.448	4.5846	**114**
115	92.000	3.6220	93.950	3.6988	115.000	4.5276	117.437	4.6235	**115**
116	92.800	3.6535	94.758	3.7306	116.000	4.5669	118.448	4.6633	**116**
117	93.600	3.6850	95.550	3.7618	117.000	4.6063	119.438	4.7023	**117**
118	94.400	3.7165	96.359	3.7937	118.000	4.6457	120.448	4.7421	**118**
119	95.200	3.7480	97.150	3.8248	119.000	4.6850	121.438	4.7810	**119**
120	96.000	3.7795	97.959	3.8566	120.000	4.7244	122.449	4.8208	**120**
121	96.800	3.8110	98.751	3.8878	121.000	4.7638	123.438	4.8598	**121**
122	97.600	3.8425	99.559	3.9197	122.000	4.8031	124.449	4.8996	**122**
123	98.400	3.8740	100.351	3.9508	123.000	4.8425	125.439	4.9385	**123**
124	99.200	3.9055	101.159	3.9826	124.000	4.8819	126.449	4.9783	**124**
125	100.000	3.9370	101.951	4.0138	125.000	4.9213	127.439	5.0173	**125**
126	100.800	3.9685	102.759	4.0456	126.000	4.9606	128.449	5.0571	**126**
127	101.600	4.0000	103.552	4.0768	127.000	5.0000	129.440	5.0960	**127**
128	102.400	4.0315	104.360	4.1086	128.000	5.0394	130.450	5.1358	**128**
129	103.200	4.0630	105.152	4.1398	129.000	5.0787	131.440	5.1748	**129**
130	104.000	4.0945	105.960	4.1716	130.000	5.1181	132.450	5.2146	**130**
131	104.800	4.1260	106.752	4.2028	131.000	5.1575	133.440	5.2536	**131**
132	105.600	4.1575	107.560	4.2346	132.000	5.1969	134.450	5.2933	**132**
133	106.400	4.1890	108.353	4.2659	133.000	5.2362	135.441	5.3323	**133**
134	107.200	4.2205	109.160	4.2976	134.000	5.2756	136.450	5.3721	**134**
135	108.000	4.2520	109.953	4.3289	135.000	5.3150	137.441	5.4111	**135**
136	108.800	4.2835	110.760	4.3606	136.000	5.3543	138.450	5.4508	**136**
137	109.600	4.3150	111.553	4.3919	137.000	5.3937	139.441	5.4898	**137**
138	110.400	4.3465	112.360	4.4236	138.000	5.4331	140.451	5.5296	**138**
139	111.200	4.3780	113.153	4.4549	139.000	5.4724	141.442	5.5686	**139**
140	112.000	4.4094	113.961	4.4866	140.000	5.5118	142.451	5.6083	**140**
141	112.800	4.4409	114.754	4.5179	141.000	5.5512	143.442	5.6473	**141**
142	113.600	4.4724	115.561	4.5496	142.000	5.5906	144.451	5.6870	**142**
143	114.400	4.5039	116.354	4.5809	143.000	5.6299	145.442	5.7261	**143**
144	115.200	4.5354	117.161	4.6126	144.000	5.6693	146.451	5.7658	**144**
145	116.000	4.5669	117.954	4.6439	145.000	5.7087	147.443	5.8048	**145**
146	116.800	4.5984	118.761	4.6756	146.000	5.7480	148.451	5.8445	**146**
147	117.600	4.6299	119.554	4.7069	147.000	5.7874	149.443	5.8836	**147**
148	118.400	4.6614	120.361	4.7386	148.000	5.8268	150.451	5.9233	**148**
149	119.200	4.6929	121.155	4.7699	149.000	5.8661	151.443	5.9623	**149**
150	120.000	4.7244	121.961	4.8016	150.000	5.9055	152.452	6.0020	**150**
151	120.800	4.7559	122.755	4.8329	151.000	5.9449	153.443	6.0411	**151**
152	121.600	4.7874	123.561	4.8646	152.000	5.9843	154.452	6.0808	**152**
153	122.400	4.8189	124.355	4.8959	153.000	6.0236	155.444	6.1198	**153**
154	123.200	4.8504	125.162	4.9276	154.000	6.0630	156.452	6.1595	**154**
155	124.000	4.8819	125.955	4.9589	155.000	6.1024	157.444	6.1986	**155**
156	124.800	4.9134	126.762	4.9906	156.000	6.1417	158.452	6.2383	**156**
157	125.600	4.9449	127.555	5.0219	157.000	6.1811	159.444	6.2773	**157**
158	126.400	4.9764	128.362	5.0536	158.000	6.2205	160.452	6.3170	**158**
159	127.200	5.0079	129.156	5.0849	159.000	6.2598	161.444	6.3561	**159**
160	128.000	5.0394	129.962	5.1166	160.000	6.2992	162.452	6.3958	**160**
161	128.800	5.0709	130.756	5.1479	161.000	6.3386	163.445	6.4348	**161**
162	129.600	5.1024	131.562	5.1796	162.000	6.3780	164.453	6.4745	**162**
163	130.400	5.1339	132.356	5.2109	163.000	6.4173	165.445	6.5136	**163**
164	131.200	5.1654	133.162	5.2426	164.000	6.4567	166.453	6.5533	**164**
165	132.000	5.1969	133.956	5.2739	165.000	6.4961	167.445	6.5923	**165**
166	132.800	5.2283	134.762	5.3056	166.000	6.5354	168.453	6.6320	**166**
167	133.600	5.2598	135.556	5.3369	167.000	6.5748	169.445	6.6711	**167**
168	134.400	5.2913	136.362	5.3686	168.000	6.6142	170.453	6.7107	**168**
169	135.200	5.3228	137.157	5.3999	169.000	6.6535	171.446	6.7498	**169**
170	136.000	5.3543	137.962	5.4316	170.000	6.6929	172.453	6.7895	**170**
171	136.800	5.3858	138.757	5.4629	171.000	6.7323	173.446	6.8286	**171**
172	137.600	5.4173	139.563	5.4946	172.000	6.7717	174.453	6.8682	**172**
173	138.400	5.4488	140.357	5.5259	173.000	6.8110	175.446	6.9073	**173**
174	139.200	5.4803	141.163	5.5576	174.000	6.8504	176.453	6.9470	**174**
175	140.000	5.5118	141.957	5.5889	175.000	6.8898	177.446	6.9861	**175**
176	140.800	5.5433	142.763	5.6206	176.000	6.9291	178.453	7.0257	**176**
177	141.600	5.5748	143.557	5.6519	177.000	6.9685	179.446	7.0648	**177**
178	142.400	5.6063	144.363	5.6836	178.000	7.0079	180.454	7.1045	**178**
179	143.200	5.6378	145.157	5.7149	179.000	7.0472	181.447	7.1436	**179**
180	144.000	5.6693	145.963	5.7466	180.000	7.0866	182.454	7.1832	**180**
181	144.800	5.7008	146.758	5.7779	181.000	7.1260	183.447	7.2223	**181**
182	145.600	5.7323	147.563	5.8096	182.000	7.1654	184.454	7.2620	**182**
183	146.400	5.7638	148.358	5.8409	183.000	7.2047	185.447	7.3011	**183**
184	147.200	5.7953	149.163	5.8726	184.000	7.2441	186.454	7.3407	**184**
185	148.000	5.8268	149.958	5.9039	185.000	7.2835	187.447	7.3798	**185**
186	148.800	5.8583	150.763	5.9356	186.000	7.3228	188.454	7.4194	**186**
187	149.600	5.8898	151.558	5.9668	187.000	7.3622	189.447	7.4586	**187**
188	150.400	5.9213	152.363	5.9986	188.000	7.4016	190.454	7.4982	**188**
189	151.200	5.9528	153.158	6.0298	189.000	7.4409	191.448	7.5373	**189**
190	152.000	5.9843	153.963	6.0615	190.000	7.4803	192.454	7.5769	**190**
191	152.800	6.0157	154.763	6.0930	191.000	7.5197	193.454	7.6163	**191**
192	153.600	6.0472	155.563	6.1245	192.000	7.5591	194.454	7.6557	**192**
193	154.400	6.0787	156.364	6.1560	193.000	7.5984	195.454	7.6951	**193**
194	155.200	6.1102	157.164	6.1875	194.000	7.6378	196.454	7.7344	**194**
195	156.000	6.1417	157.964	6.2190	195.000	7.6772	197.454	7.7738	**195**
196	156.800	6.1732	158.764	6.2505	196.000	7.7165	198.455	7.8132	**196**
197	157.600	6.2047	159.564	6.2820	197.000	7.7559	199.455	7.8525	**197**
198	158.400	6.2362	160.364	6.3135	198.000	7.7953	200.455	7.8919	**198**
199	159.200	6.2677	161.164	6.3450	199.000	7.8346	201.455	7.9313	**199**
200	160.000	6.2992	161.964	6.3765	200.000	7.8740	202.455	7.9707	**200**
201	160.800	6.3307	162.759	6.4078	201.000	7.9134	203.449	8.0098	**201**
202	161.600	6.3622	163.559	6.4393	202.000	7.9528	204.449	8.0492	**202**
203	162.400	6.3937	164.359	6.4708	203.000	7.9921	205.449	8.0885	**203**
204	163.200	6.4252	165.159	6.5023	204.000	8.0315	206.449	8.1279	**204**
205	164.000	6.4567	165.959	6.5338	205.000	8.0709	207.449	8.1673	**205**
240	192.000	7.5591	193.965	7.6364	240.000	9.4488	242.456	9.5455	**240**
280	224.000	8.8189	225.966	8.8963	280.000	11.0236	282.457	11.1204	**280**
300	240.000	9.4488	241.966	9.5262	300.000	11.8110	302.458	11.9078	**300**
340	272.000	10.7087	273.967	10.7861	340.000	13.3858	342.459	13.4826	**340**
380	304.000	11.9685	305.967	12.0460	380.000	14.9606	382.459	15.0575	**380**
400	320.000	12.5984	321.968	12.6759	400.000	15.7480	402.460	15.8449	**400**
440	352.000	13.8583	353.968	13.9357	440.000	17.3228	442.460	17.4197	**440**
480	384.000	15.1181	385.968	15.1956	480.000	18.8976	482.460	18.9945	**480**
500	400.000	15.7480	401.968	15.8255	500.000	19.6850	502.461	19.7819	**500**

SECTION 11 CONTACT RATIO

To assure continuous smooth tooth action, as one pair of teeth ceases action a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as is possible. A measure of this overlap action is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. **Figure 11-1** shows the geometry for a spur gear pair, which is the simplest case, and is representative of the concept for all gear types. The length-of-action is determined from the intersection of the line-of-action and the outside radii. The ratio of the length-of-action to the base pitch is determined from:

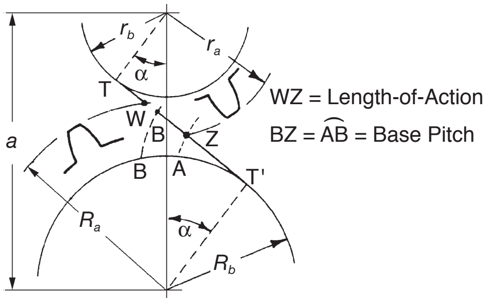


Fig. 11-1 Geometry of Contact Ratio

$$\varepsilon_\gamma = \frac{\sqrt{(R_a^2 - R_b^2)} + \sqrt{(r_a^2 - r_b^2)} - a\sin\alpha}{\pi m \cos\alpha} \qquad \textbf{(11-1)}$$

It is good practice to maintain a contact ratio of 1.2 or greater. Under no circumstances should the ratio drop below 1.1, calculated for all tolerances at their worst case values.

A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact. Such a high ratio is generally not obtained with external spur gears, but can be developed in the meshing of internal gears, helical gears, or specially designed nonstandard external spur gears.

When considering all types of gears, contact ratio is composed of two components:

1. Radial contact ratio (plane of rotation perpendicular to axes), ε_α
2. Overlap contact ratio (axial), ε_β

The sum is the total contact ratio, ε_γ.

The overlap contact ratio component exists only in gear pairs that have helical or spiral tooth forms.

11.1 Radial Contact Ratio Of Spur And Helical Gears, ε_α

The equations for radial (or plane of rotation) contact ratio for spur and helical gears are given in **Table 11-1**, with reference to **Figure 11-2**.

When the contact ratio is inadequate, there are three means to increase it. These are somewhat obvious from examination of **Equation (11-1)**.

1. Decrease the pressure angle. This makes a longer line-of-action as it extends through the region between the two outside radii.

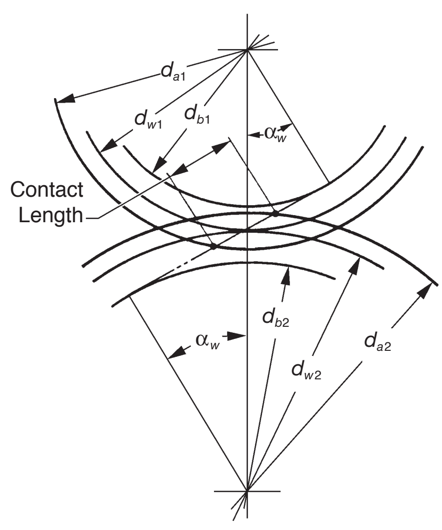


Fig. 11-2 Radial Contact Ratio of Parallel Axes Gear ε_α

Table 11-1 Equations of Radial Contact Ratio on Parallel Axes Gear, ε_α

Type of Gear Mesh		Formula of Radial Contact Ratio, ε_α
Spur Pair	Gear ① Gear ②	$\dfrac{\sqrt{(\frac{d_{a1}}{2})^2-(\frac{d_{b1}}{2})^2}+\sqrt{(\frac{d_{a2}}{2})^2-(\frac{d_{b2}}{2})^2}-a_x\sin\alpha_w}{\pi m\cos\alpha}$
Spur Gear and Rack	Gear ① Rack ②	$\dfrac{\sqrt{(\frac{d_{a1}}{2})^2-(\frac{d_{b1}}{2})^2}+\frac{h_{a2}-x_1 m}{\sin\alpha}-\frac{d_1}{2}\sin\alpha}{\pi m\cos\alpha}$
External and Internal Spur	External Gear ① Internal Gear ②	$\dfrac{\sqrt{(\frac{d_{a1}}{2})^2-(\frac{d_{b1}}{2})^2}-\sqrt{(\frac{d_{a2}}{2})^2-(\frac{d_{b2}}{2})^2}+a_x\sin\alpha_w}{\pi m\cos\alpha}$
Helical Pair	Gear ① Gear ②	$\dfrac{\sqrt{(\frac{d_{a1}}{2})^2-(\frac{d_{b1}}{2})^2}+\sqrt{(\frac{d_{a2}}{2})^2-(\frac{d_{b2}}{2})^2}-a_x\sin\alpha_{wt}}{\pi m_t\cos\alpha_t}$

2. Increase the number of teeth. As the number of teeth increases and the pitch diameter grows, again there is a longer line-of-action in the region between the outside radii.
3. Increase working tooth depth. This can be done by adding addendum to the tooth and thus increase the outside radius. However, this requires a larger dedendum, and requires a special tooth design.

An example of helical gear:

m_n = 3	α_n = 20°	β = 30°	z_1 = 12
z_2 = 60	x_1 = +0.09809	x_2 = 0	a_x = 125
α_t = 22.79588°	α_{wt} = 23.1126°	m_t = 3.46410	d_{a1} = 48.153
d_{a2} = 213.842	d_{b1} = 38.322	d_{b2} = 191.611	ε_α = 1.2939

Note that in **Table 11-1** only the radial or circular (plane of rotation) contact ratio is considered. This is true of both the spur and helical gear equations. However, for helical gears this is only one component of two. For the helical gear's total contact ratio, ε_γ, the overlap (axial) contact ratio, ε_β, must be added. See **Paragraph 11.4**.

11.2 Contact Ratio Of Bevel Gears, ε_α

The contact ratio of a bevel gear pair can be derived from consideration of the eqivalent spur gears, when viewed from the back cone. See **Figure 8-8**.

With this approach, the mesh can be treated as spur gears. **Table 11-2** presents equations calculating the contact ratio.

An example of spiral bevel gear (see **Table 11-2**):

m = 3	α_n = 20°	β = 35°	z_1 = 20
z_2 = 40	α_t = 23.95680°	d_1 = 60	d_2 = 120
R_{v1} = 33.54102	R_{v2} = 134.16408	R_{vb1} = 30.65152	R_{vb2} = 122.60610
h_{a1} = 3.4275	h_{a2} = 1.6725	R_{va1} = 36.9685	R_{va2} = 135.83658
ε_α = 1.2825			

11.3 Contact Ratio For Nonparallel And Nonintersecting Axes Pairs, ε

This group pertains to screw gearing and worm gearing. The equations are approximations by considering the worm and worm gear mesh in the plane perpendicular to worm gear axis and likening it to spur gear and rack mesh. **Table 11-3** presents these equations.

Example of worm mesh:

m_x = 3	α_n = 20°	z_w = 2	z_2 = 30
d_1 = 44	d_2 = 90	γ = 7.76517°	α_x = 20.17024°
h_{a1} = 3	d_{th} = 96	d_{b2} = 84.48050	ε = 1.8066

11.4 Axial (Overlap) Contact Ratio, ε_β

Helical gears and spiral bevel gears have an overlap of tooth action in the axial direction. This overlap adds to the contact ratio. This is in contrast to spur gears which have no tooth action in the axial direction. Thus, for the same tooth proportions in the plane of rotation, helical and spiral bevel gears offer a significant increase in contact ratio. The magnitude of axial contact ratio is a direct function of the gear width, as illustrated in **Figure 11-3**. Equations for calculating axial contact ratio are presented in **Table 11-4**.

It is obvious that contact ratio can be increased by either increasing the gear width or increasing the helix angle.

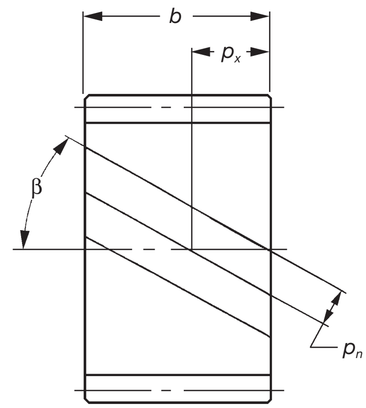


Fig. 11-3 Axial (Overlap) Contact Ratio

Table 11-2 Equations for Contact Ratio for a Bevel Gear Pair

Item	Symbol	Equation for Contact Ratio	
Back Cone Distance	R_v	$\frac{d}{2\cos\delta}$	
Base Circle Radius of an Equivalent Spur Gear	R_{vb}	Straight Bevel Gear $R_v\cos\alpha$	Spiral Bevel Gear $R_v\cos\alpha_t$
Outside Radius of an Equivalent Spur Gear	R_{va}	$R_v + h_a$	
Contact Ratio	ε_α	Straight Bevel Gear $\frac{\sqrt{R_{va1}^2-R_{vb1}^2}+\sqrt{R_{va2}^2-R_{vb2}^2}-(R_{v1}+R_{v2})\sin\alpha}{\pi m\cos\alpha}$	
		Spiral Bevel Gear $\frac{\sqrt{R_{va1}^2-R_{vb1}^2}+\sqrt{R_{va2}^2-R_{vb2}^2}-(R_{v1}+R_{v2})\sin\alpha_t}{\pi m\cos\alpha_t}$	

Table 11-3 Equations for Contact Ratio of Nonparallel and Nonintersecting Meshes

Type of Gear Mesh	Equation of Contact Ratio, ε
Screw Gear ① Screw Gear ②	$\frac{\sqrt{\left(\frac{d_{a1}}{2}\right)^2-\left(\frac{d_{b1}}{2}\right)^2}+\sqrt{\left(\frac{d_{a2}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}-\frac{a-\frac{d_{b1}\cos\alpha_{t1}}{2}-\frac{d_{b2}\cos\alpha_{t2}}{2}}{\sin\alpha_n}}{\pi m_n\cos\alpha_n}$
Worm ① Worm Gear ②	$\frac{\frac{h_{a1}-x_{x2}m_x}{\sin\alpha_x}+\sqrt{\left(\frac{d_{th}}{2}\right)^2-\left(\frac{d_{b2}}{2}\right)^2}-\frac{d_2}{2}\sin\alpha_x}{\pi m_x\cos\alpha_x}$

Table 11-4 Equations for Axial Contact Ratio of Helical and Spiral Bevel Gears, ε_β

Type of Gear	Equation of Contact Ratio	Example
Helical Gear	$\frac{b\sin\beta}{\pi m_n}$	$b = 50$, $\beta = 30°$, $m_n = 3$ $\varepsilon_\beta = 2.6525$
Spiral Bevel Gear	$\frac{R_e}{R_e - 0.5b}\ \frac{b\tan\beta_m}{\pi m}$	From **Table 8-6**: $R_e = 67.08204$, $b = 20$, $\beta_m = 35°$, $m = 3$, $\varepsilon_\beta = 1.7462$

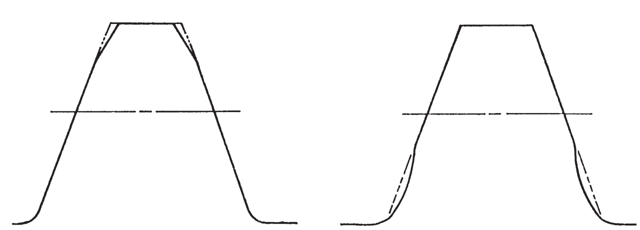
NOTE: The module m in spiral bevel gear equation is the normal module.

SECTION 12 GEAR TOOTH MODIFICATIONS

Intentional deviations from the involute tooth profile are used to avoid excessive tooth load deflection interference and thereby enhances load capacity. Also, the elimination of tip interference reduces meshing noise. Other modifications can accommodate assembly misalignment and thus preserve load capacity.

12.1 Tooth Tip Relief

There are two types of tooth tip relief. One modifies the addendum, and the other the dedendum. See **Figure 12-1**. Addendum relief is much more popular than dedendum modification.



(a) Addendum Tip Relief **(b)** Dedendum Modification

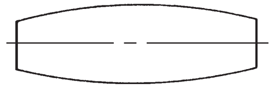
Fig. 12-1 Tip Relief

12.2 Crowning And Side Relieving

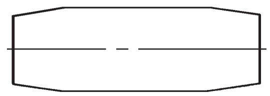
Crowning and side relieving are tooth surface modifications in the axial direction. See **Figure 12-2**.

Crowning is the removal of a slight amount of tooth from the center on out to reach edge, making the tooth surface slightly convex. This method allows the gear to maintain contact in the central region of the tooth and permits avoidance of edge contact with consequent lower load capacity. Crowning also allows a greater tolerance in the misalignment of gears in their assembly, maintaining central contact.

Relieving is a chamfering of the tooth surface. It is similar to crowning except that it is a simpler process and only an approximation to crowning. It is not as effective as crowning.

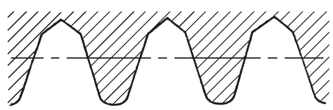


(a) Crowning

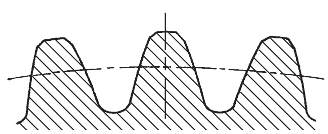


(b) Side Relieving

Fig. 12-2 Crowning and Relieving



(a) Teeth Form of Semitopping Cutter



(b) Semitopped Teeth Form

Fig. 12-3 Semitopping Cutter and the Gear Profile Generated

12.3 Topping And Semitopping

In topping, often referred to as top hobbing, the top or outside diameter of the gear is cut simultaneously with the generation of the teeth. An advantage is that there will be no burrs on the tooth top. Also, the outside diameter is highly concentric with the pitch circle. This permits secondary machining operations using this diameter for nesting.

Semitopping is the chamfering of the tooth's top corner, which is accomplished simultaneously with tooth generation. **Figure 12-3** shows a semitopping cutter and the resultant generated semitopped gear. Such a tooth tends to prevent corner damage. Also, it has no burr. The magnitude of semitopping should not go beyond a proper limit as otherwise it would significantly shorten the addendum and contact ratio. **Figure 12-4** specifies a recommended magnitude of semitopping.

Both modifications require special generating tools. They are independent modifications but, if desired, can be applied simultaneously.

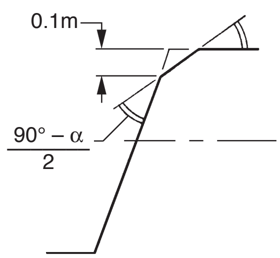


Fig. 12-4 Recommended Magnitude of Semitopping

SECTION 13 GEAR TRAINS

The objective of gears is to provide a desired motion, either rotation or linear. This is accomplished through either a simple gear pair or a more involved and complex system of several gear meshes. Also, related to this is the desired speed, direction of rotation and the shaft arrangement.

13.1 Single-Stage Gear Train

A meshed gear is the basic form of a single-stage gear train. It consists of z_1 and z_2 numbers of teeth on the driver and driven gears, and their respective rotations, n_1 & n_2. The speed ratio is then:

$$\text{speed ratio} = \frac{z_1}{z_2} = \frac{n_2}{n_1} \qquad \textbf{(13-1)}$$

13.1.1 Types Of Single-Stage Gear Trains

Gear trains can be classified into three types:

1. Speed ratio > 1, increasing: $n_1 < n_2$
2. Speed ratio =1, equal speeds: $n_1 = n_2$
3. Speed ratio < 1, reducing: $n_1 > n_2$

Figure 13-1 illustrates four basic types. For the very common cases of spur and bevel meshes, **Figures 13-1(a)** and **13-1(b)**, the direction of rotation of driver and driven gears are reversed. In the case of an internal gear mesh, **Figure 13-1(c)**, both gears have the same direction of rotation. In the case of a worm mesh, **Figure 13-1(d)**, the rotation direction of z_2 is determined by its helix hand.

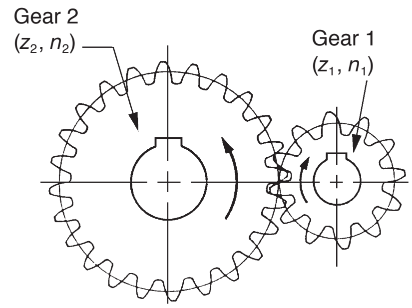
In addition to these four basic forms, the combination of a rack and gear can be considered a specific type. The displacement of a rack, l, for rotation θ of the mating gear is:

$$l = \frac{\pi m z_1 \theta}{360} \qquad \textbf{(13-2)}$$

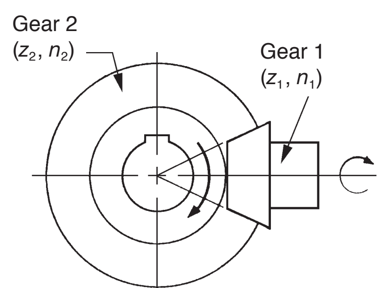
where:

$\pi\, m$ is the standard circular pitch

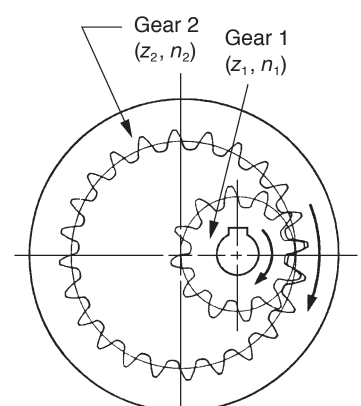
z_1 is the number of teeth of the gear



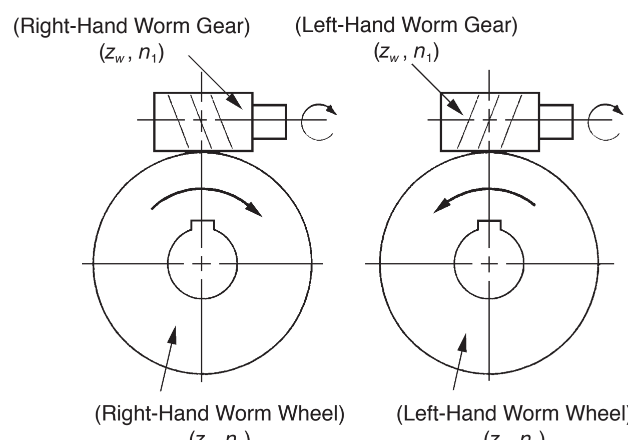
(a) A Pair of Spur Gears



(b) Bevel Gears



(c) Spur Gear and Internal Gear



(d) Worm Mesh

Fig. 13-1 Single-Stage Gear Trains